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COMPACT INTEGRAL DOMAINS

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It is well known that if A is a compact integral domain and R is its Jacobson radical, then $A = R$ or A/R is a division ring and A has an identity. The object of this paper is to investigate some of the algebraic properties of A.

If A has an identity and finite characteristic, then there exists a maximal subfield F of A which is isomorphic to A/R. Furthermore A is topologically isomorphic to $F + R$. The existence of a subfield is a necessary and sufficient condition for A to have finite characteristic. If A does not have an identity but does have finite characteristic, then it can be openly embedded in a compact integral domain with an identity. Finally, the main result shows that if the center of A is open, then A is commutative.

1. Preliminaries. An integral domain is defined to be a ring with more than one element such that the nonzero elements form a multiplicative semigroup (which is not necessarily commutative). It is always assumed that a topological ring is Hausdorff.

Throughout this paper A will denote a compact integral domain and R will denote its Jacobson radical.

It has been shown [4, Lemma 2, p. 279] that A has a fundamental system of neighborhoods at zero consisting of open (two-sided) ideals. Kaplansky has shown that R is open [3, Th. 7, p. 161]. Also, that $A = R$ or A/R is a division ring and A has an identity [3, Th. 19, p. 168].

Because A is an integral domain, it can have no elements or ideals that are nilpotent in an algebraic sense, but nilpotency can be defined in a topological sense. We say that an element x is nilpotent if $\lim x^n = 0$, and that an ideal V is nilpotent if for every open set containing zero, there is an integer N such that for every $n \geqq N$, $V^n \subset W$ where V^n is the set of all finite sums of the product of n elements of V.

It has been shown that R is nilpotent [3, Th. 14, p. 163].

The following theorem, which may be thought of as an extension of Wedderburn's theorem, immediately follows from the above results.

THEOREM 1. *Any compact semi-simple integral domain is a finite field.*