ON THE BARYCENTRIC HOMOMORPHISM IN A SINGULAR COMPLEX

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INTRODUCTION

0.1. Radó has introduced and studied the following approach to singular homology theory (see [2; 3; 4] for details). With a general topological space X associate a complex $R = R(X)$ in the following manner. For integers $p \geq 0$, let $v_0, \cdots, v_p$ be a sequence of $p+1$ points in Hilbert space E_∞, which are not required to be distinct or linearly independent, and let $|v_0, \cdots, v_p|$ denote their convex hull. Suppose that T is a continuous mapping from $|v_0, \cdots, v_p|$ into X. Then the sequence $v_0, \cdots, v_p$ jointly with T determines a p-cell in R, which is denoted by $(v_0, \cdots, v_p, T)^R$. The free Abelian group C_p^R generated by the p-cells in R is termed the group of integral p-chains in R. For integers $p < 0$, C_p^R is defined to be the group consisting of the zero element alone. The boundary operator $\partial_p^R \colon C_p^R \longrightarrow C_{p-1}^R$ is defined, in the usual manner, as the trivial homomorphism if $p \leq 0$, and by the relation

$$\partial_p^R (v_0, \cdots, v_p, T)^R = \sum_{i=0}^{p} (-1)^p (v_0, \cdots, \hat{v}_i, \cdots, v_p, T)^R$$

if $p > 0$. Since $\partial_{p-1}^R \partial_p^R = 0$, one introduces the subgroup Z_p^R of p-cycles in C_p^R and the subgroup B_p^R of p-boundaries in C_p^R in the customary way, and defines the quotient group of Z_p^R with respect to B_p^R to be the homology group H_p^R.

0.2. The approach to singular homology theory pursued by Radó differs from other approaches in that absolutely no identifications are made. Thus two p-cells $(v_0', \cdots, v_p', T')^R$ and $(v_0'', \cdots, v_p'', T'')^R$ are equal only if they are identical; that is, if $v_i' = v_i''$ for $i = 0, \cdots, p$ and $T' \equiv T''$ on $|v_0', \cdots, v_p'| = |v_0'', \cdots, v_p''|$. In [3; 4], Radó introduces a technique for making identifications in a general Mayer complex and applies his procedure to study identifications in R, particularly those which yield homology groups isomorphic to the H_p^R. It is a primary purpose of the present paper to pursue the matter further in

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