# WEAK AND STRONG LIMITS OF SPECTRAL OPERATORS

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The present paper is a contribution to the theory of spectral operators in Banach spaces developed by N. Dunford in [8] and [9]. A bounded operator $S$ is a *spectral operator of scalar type* if, roughly speaking, it has a representation

$$S = \int_{\sigma(S)} \lambda E(d\lambda)$$

where $E(\cdot)$ is a resolution of the identity similar to that possessed by a normal operator in Hilbert space. The initial problem we are concerned with is to find conditions under which a weak or strong limit of scalar type spectral operators is again in this class. The results are then applied to the study of certain weakly closed algebras of spectral operators.

Section 1 contains a brief summary of definitions and results from [8] and [9]. In §2 conditions are found under which a strong limit of scalar type spectral operators is a scalar type spectral operator, the principal restriction imposed in the limiting operators being on the nature of their spectra. The operators need not commute.

Suppose that the underlying space $\mathfrak{X}$ is reflexive. If $\mathfrak{A}$ is an algebra generated by a bounded Boolean algebra $\mathfrak{B}$ of projections, then by a theorem of Dunford [9], each operator in $\mathfrak{A}$ is a scalar type spectral operator. We show (Theorem 4.1) that every operator in the weak closure $\mathfrak{W}$ of $\mathfrak{A}$ is a scalar type operator, and characterize $\mathfrak{W}$ as the algebra generated (in the uniform topology) by the strong closure of $\mathfrak{B}$. The principal tool used is the equivalence (due to Dunford [7]) of strong closure and lattice completeness for bounded Boolean algebras of projections. We give a new proof of this theorem.

The paper concludes with a characterization of the weakly closed algebra generated by a single scalar type spectral operator with real spectrum. Our proof of this theorem gives a more direct proof of the corresponding result of Segal [22] for Hilbert space.

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