UNBOUNDED SPECTRAL OPERATORS

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1. Introduction. Our purpose in the present paper is to study the structure and operational calculus of unbounded spectral operators. Bounded spectral operators have been introduced and studied by N. Dunford in [2] and [3], and the present paper is an investigation in the unbounded case of certain of the results of [3]. Interest in the abstract theory of unbounded spectral operators arises from important results of J. Schwartz [7], who has shown that the members of a large class of differential operators on a finite interval determine unbounded spectral operators in Hilbert space.

Let $\mathfrak{B}$ denote the Borel subsets of the complex plane, and let $\mathfrak{X}$ be a complex Banach space. We shall call a mapping E from $\mathfrak{B}$ to projection operators in $\mathfrak{X}$ a *resolution of the identity* if it is a homomorphism. That is,

$$E(e)E(f) = E(ef), \qquad E(e) \cup E(f) = E(e \cup f), \qquad e, f \in \mathfrak{B}$$

$$E(e') = I - E(e), \qquad E(\phi) = 0, \qquad E(p) = I, \qquad e \in \mathfrak{B};$$

$E(e)$ is bounded,

$$|E(e)| \leq M, \qquad e \in \mathfrak{B};$$

and[1] the vector-valued set function $E(e)x$ is countably additive. Here ϕ is the void set, p the plane, and e' the complement of e in p.

A closed operator T will be called a *spectral operator* if there is a resolution of the identity E such that:

(1) The domain $D(T)$ of T contains the dense subspace $\mathfrak{X}_0 = \{x \mid x = E(\sigma)x, \sigma \in \mathfrak{B}, \sigma \text{ bounded}\}$.

(2) If $\sigma \in \mathfrak{B}$, $E(\sigma)D(T) \subset D(T)$ and $E(\sigma)Tx = TE(\sigma)x$, $x \in D(T)$.

[1] The last condition is somewhat more restrictive than in [3].

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