ON THE DIVISIBILITY OF THE CLASS NUMBER OF QUADRATIC FIELDS

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1. Introduction. It is well known that there exist infinitely many quadratic extensions of the rationals each with class number divisible by 2. In fact, if the discriminant of the field contains more than two prime factors, then 2 divides the class number. Max Gut [1] generalized this result to show that there exist infinitely many quadratic imaginary fields each with class number divisible by 3. In this present paper we prove that there exist infinitely many quadratic imaginary fields each with class number divisible by g where g is any given rational integer.

The method extends to yield certain results about quadratic real fields, but these are not as sharp as on quadratic imaginary fields.

2. Theorem. In the following we may assume without loss of generality that g is positive, sufficiently large, and even.

LEMMA 1. *Denote by N the number of square-free integers of the form*

$$3^g - x^2, \text{ where } 2 \mid x,\ 0 < x < (2.3^{g-1})^{1/2}.$$

Then, for g sufficiently large,

$$N \geq \frac{1}{25}\ 3^{g/2}.$$

Proof. Denote by d the expression

$$d = 3^g - x^2, \tag{1}$$

where

$$2 \mid x, \quad 0 < x < (2 \cdot 3^{g-1})^{1/2}. \tag{2}$$

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