HOMOGENEOUS STOCHASTIC PROCESSES*

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Summary. The form of a stationary translation-invariant Markov process on the real line has been known for some time, and these processes have been variously characterized as infinitely divisible or infinitely decomposable. The purpose of this paper is to study a natural generalization of these processes on a homogeneous space (X, G). Aside from the lack of structure inherent in the very generality of the spaces (X, G), the basic obstacles to be surmounted stem from the presence of non trivial compact subgroups in G and the non commutativity of G, which precludes the use of an extended Fourier analysis of characteristic functions, a tool which played a dominant role in the classical studies. Even in the general situation there is a striking similarity between homogeneous processes and their counterparts on the real line.

A homogeneous process is a process in the terminology of Feller [3] on a locally compact Hausdorff space X, whose transition probabilities $P(t, x, dy)$ are invariant under the action of elements $g \in G$ of a transitive group of homeomorphisms of X, in the sense that $P(t, g[x], g[dy]) = P(t, x, dy)$. It is shown that if every compact subset of X is separable or G is commutative the family of measures $t^{-1}P(t, x, \cdot)$ converges to a not necessarily bounded Borel measure $Q_x(\cdot)$ on $X\text{-}\{x\}$ as $t \to 0$, meaning that for every bounded continuous, complex valued function f on X which vanishes in a neighborhood of x and is constant at infinity $t^{-1}P(t, x, f) \to Q_x(f)$.

In 3 we show that the paths of a separable homogeneous process are bounded on every bounded t-interval and have right and left limits at every t with probability one. If the action of G on X is used to translate the origin of each jump to x, it is shown for suitably regular compact sets D that the probability of a jump into D while $t \in [0, T]$ is given by 1–exp $\{-TQ_x(D)\}$. The maps $f \to P(t, \cdot, f) = (T_t f)(\cdot)$ map the Banach space, $C(X)$, of continuous functions generated by the constants and functions with compact support into itself, and by a suitable normalization can be assumed strongly continuous for $t \geq 0$. Indeed, T_t is a strongly continuous semi-group. The domain $D(A)$ of the infinitesimal generator A of T_t admits a smoothing operation whose precise

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