ON THE BREADTH AND CO-DIMENSION OF A TOPOLOGICAL LATTICE

LEE W. ANDERSON

Consider the following two conjectures:

Conjecture 1. (E. Dyer and A. Shields [7]) If L is a compact, connected, metrizible, distributive topological lattice then $\dim(L) =$ breadth of L.

Conjecture 2. (A. D. Wallace [10]) If L is a compact, connected topological lattice and if $\dim(L) = n$ then the center of L contains at most $2^n - 2$ elements.

The purpose of this note is to prove the following results:

(1) If L is a locally compact distributive topological lattice and if each pair of comparable points is contained in a closed connected chain then the breadth of $L \leqq$ codim (L).

(2) If L is a compact, connected, distributive topological lattice and if $\operatorname{codim}(L) \leqq n$ then the center of L contains at most $2^n - 2$ elements.

1. NOTATION. The terminology and notation used in this paper is the same as in [1] [2] and [3]. If L is a lattice, then the *breadth of* L [4], hereafter denoted by $Br(L)$, is the smallest integer n such that any finite subset, F, of L has a subset F' of at most n elements such that $\inf(F) = \inf(F')$.

If A is a subset of a lattice, let $\wedge A^n$ denote the set of all elements of the form $x_1 \wedge x_2 \wedge \cdots \wedge x_n$ where $x_i \in A$.

2. $Br(L) \leqq cd(L)$. The proof of the following lemma is quite straight forward and will be omitted.

LEMMA 1. *If L is a lattice then the following are equivalent:*

(i) $Br(L) \leqq n$

(ii) *If A is an $n + 1 -$ element subset of L then A contains an n-element subset B, such that* $\inf(A) = \inf(B)$.

(iii) *If A is a subset of L and if $m, p \geqq n$ then* $\wedge A^m = \wedge A^p$.

If L is a topological lattice, then L is *chain-wise connected* if for each pair of elements, x and y, in L with $x \leqq y$ there is a closed connected chain from x to y. Clearly a compact connected topological lattice is chainwise connected.

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