UNIFORM NEIGHBORHOOD RETRACTS

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Introduction. This paper is a systematic investigation of a part of the mapping theory of uniform spaces. The central concept through most of the paper is that of an *ANRU* as defined in [15]; that is, a uniform space X which, whenever it is embedded in a uniform space Y, is a uniformly continuous retract of a uniform neighborhood of itself. Roughly, the first third of the paper treats the construction of *ANRU*'s; the middle part treats necessary conditions for *ANRU*'s and the last part treats some uses of *ANRU*'s.

In compact spaces (and to a great extent in paracompact spaces) there is a well-developed theory analyzing the general space X in terms of its mappings into polyhedra; in several strong senses, there are sufficiently many mappings into polyhedra. There are enough mappings even into a closed interval to determine the topology. There is also the program proposed by M. H. Stone [29] of analyzing X in terms of mappings of Boolean spaces onto X; this is sufficient in principle, since every compact space is a quotient space of a Boolean space, but it seems to be very difficult to work out and in any case it cannot generalize much beyond the compact spaces. Note that the polyhedra are suitable for mapping into; they are *ANR*'s, and a large (sufficiently large) class of them are *AR*'s or *injective objects*. Gleason has shown [9] that a large class of Boolean spaces are projective objects in the category of compact spaces, but there remains the fundamental difficulty that the Boolean spaces themselves are not well known.

Among uniform spaces, finite-dimensional polyhedra are known to be *ANRU*'s [15]; but there are not nearly enough of them. The basic difficulty here is that there are not enough mappings of uniform space X into the real line, in the sense that knowing all of them does not suffice to determine the uniformity of X. The simple observation which generates Part I of this paper is that if we know which real-valued functions on X are uniformly continuous, and which families of real-valued functions on X are equiuniformly continuous, this does suffice to determine the uniformity of X. Restated: every uniform space can be embedded in a product of spaces $U(D_\alpha, R)$ of all real-valued (uniformly continuous) functions on discrete spaces D_α. As it happens, such a product space need not be an *ANRU*; but this can be changed by using a closed interval I in place of R.

Section 1 gives the argument just sketched to prove that every

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