SEMICHARACTERS OF THE CARTESIAN PRODUCT OF TWO SEMIGROUPS

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1. If S and T are semigroups, then by $S \times T$ we mean the semigroup consisting of the Cartesian product $S \times T$ of the sets S and T with coordinatewise multiplication. The semigroup $S \times T$ is called the *Cartesian product of the seimgroups* S and T. A complex-valued multiplicative function on a semigroup S is called a *semicharacter* of S if it is different from 0 at some point and is bounded (1.3, [1]). The set of all semicharacters of S is denoted by $\hat{S}$.

We show that $\widehat{S \times T} = \{\chi | \chi(x, u) = \phi(x)\psi(u) \text{ for some } \phi \in \hat{S}, \psi \in \hat{T}\}$ (2.4). We obtain a similar result for continuous semicharacters of topological semigroups (3.3). One of the most interesting consequences of the above results is a theorem on prime ideals (2.6). A subset I of a semigroup S is called a *prime ideal* of S if I is a proper (i.e., $\neq S$) two-sided ideal of S whose complement in S is a semigroup. For convenience we also call the empty set a prime ideal (cf. Definitions 2, 2a, [2]). We also prove a theorem concerning continuity of the semicharacters of the Cartesian product $S \times T$ of two topological semigroups (3.4).

If A and B are sets, then $A - B$ will denote the set of all elements of A which are not contained in B. A semigroup will always be nonempty. A nonempty subset I of S is said to be an (two-sided) *ideal* of S if $xy, yx \in I$ for all $x \in S, y \in I$.

All results in this paper are stated for the Cartesian product of two semigroups. However, a simple inductive argument shows that all of them generalize to the Cartesian product of any finite number of semigroups.

This paper is an excerpt from the author's doctoral dissertation. The writer wishes to express his sincere gratitude to Professor Herbert S. Zuckerman for his very kind help in the preparation of this research. The writer also is indebted to Dr. K. A. Ross and Professor Edwin Hewitt for useful suggestions. This research was supported by the Office of Naval Research.

2. If S and T are semigroups with two-sided identities, then semicharacters of $S \times T$ are obtained easily from the semicharacters of S and T. (If e and f are identities of S and T, respectively, then each element (x, u) of $S \times T$ can be written as $(x, f)(e, u)$.) In 5, [3], Št. Schwarz considers this case for commutative semigroups. We first introduce two definitions.

Received March 31, 1961.