TAUBERIAN CONSTANTS FOR THE $[J, f(x)]$ TRANSFORMATIONS

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1. Introduction. Let $\{s_n\}$ $(n \geqq 0)$ $(s_n = a_0 + \cdots + a_n)$ be a sequence of real or complex numbers. Denote by $t(x)$ a linear transform T

$$t(x) = \sum_{n=0}^{\infty} c_n(x) s_n$$

of $\{s_n\}$ supposed convergent for all sufficiently large values of x. In addition to classical Abelian and Tauberian theorems which give information about one of $\lim_{x\to\infty} t(x)$ and $\lim_{n\to\infty} s_n$ when the other exists, it is possible to find estimates of

$$\lim_{n\to\infty,\ k_n\to\infty} \sup |\, t(x_n) - s_n \,|$$

when neither $\lim t(x)$ nor $\lim s_n$ is supposed to exist but $\{a_n\}$ is assumed to satisfy the condition

$$\limsup_{n\to\infty} |\, n a_n \,| < +\infty \,. \tag{1.1}$$

Such estimates were obtained first by H. Hadwiger [4] for the Abel transform $t(x)$. Delange [3] developed a general theory for such estimates when $x_n = qn$, where q is some fixed positive number. Usualy the estimates proved have the form

$$\lim_{n\to\infty,\, x_n\to\infty} \sup |\, t(x_n) - s_n \,| \leqq C. \limsup_{n\to\infty} |\, n a_n \,| \,.$$

We call the constant C a Tauberian constant associated with the transformation T.

In this paper we shall prove some Hadwiger-type inequalities for a class of $[J, f(x)]$ transformations (see § 2). In these results the connection between n and x_n will be more general than the relation $x_n = qn$.

As a consequence of the main result of this paper we shall obtain the interesting result that for any sequence $\{s_n\}$ satisfying (1.1) the set of limit points of $\{s_n\}$ and the set of limit points of the Borel transform of $\{s_n\}$ are the same set

2. The $[J, f(x)]$ transformations. The class of $[J, f(x)]$ transformations was defined in [5], where it was shown that the $[J, f(x)]$ transfor-

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