NORMAL SUBGROUPS OF MONOMIAL GROUPS

ALLAN B. GRAY, JR.

1. Introduction. Let U be the set consisting of $x_1, x_2, x_3, \cdots x_n$. Let H be a fixed group. A *monomial substitution* of U over H is a transformation of the form,

$$y = \begin{pmatrix} x_1 , & x_2 , & x_3 , \cdots, & x_n \\ h_1 x_{j_1}, & h_2 x_{i_2}, & h_3 x_{i_3}, \cdots, & h_n x_{i_n} \end{pmatrix} \begin{matrix} x_j \in U \\ h_i \in H \end{matrix}$$

where the mapping of the x's is one-to-one. The h_j are called the factors of y. If.

$$y_1 = \begin{pmatrix} x_1 , & x_2 , & x_3 , \cdots, & x_n \\ k_1 x_{j_1}, & k_2 x_{j_2}, & k_3 x_{j_3}, \cdots, & k_n x_{j_n} \end{pmatrix}$$

then

$$y y_1 = \begin{pmatrix} x_1 , & x_2 , & x_3 , & \cdots, & x_n \\ h_1 k_{i_1} x_{j_{i_1}}, & h_2 k_{i_2} x_{j_{i_2}}, & h_3 k_{i_3} x_{j_{i_3}}, & \cdots, & h_n k_{i_n} x_{j_{i_n}} \end{pmatrix} .$$

By this definition of multiplication the set of all substitutions form a group $\Sigma_n(H)$. Denote by V the set of all substitutions of the form

$$y = \begin{pmatrix} x_1 , & x_2 , & x_3 , \cdots, & x_n \\ h_1 x_1, & h_2 x_2, & h_3 x_3, \cdots, & h_n x_n \end{pmatrix} = [h_1, h_2, h_3, \cdots, h_n] .$$

Then V, called the basis group, is a normal subgroup of $\Sigma_n(H)$. A permutation is an element of the form

$$\begin{pmatrix} x_1 , & x_2 , \cdots, & x_n \\ e x_{i_1}, & e x_{i_2}, \cdots, & e x_{i_n} \end{pmatrix} = \begin{pmatrix} 1, & 2, \cdots, & n \\ i_1, & i_2, \cdots, & i_n \end{pmatrix} .$$

where e is the identity of H. Cyclic representation will also be used for elements of this type. The set S_n of all such elements is a subgroup of $\Sigma_n(H)$. Furthermore $\Sigma_n(H) = V \cup S$, $V \cap S = E$ where E is the identity of $\Sigma_n(H)$. Any element y of $\Sigma_n(H)$ can be written as $y = vs$ where $v \in V$ and $s \in S$. Ore [1] has studied this group for finite U and some of his results have been extended in [2] and [3].

The normal subgroups of $\Sigma_n(H) = \Sigma_n$ for U a finite set have been determined in [1]. The normal subgroups for $o(U) = B = \aleph_u$, $u \geqq 0$, where $o(U)$ means the number of elements of U, have been determined for rather general cases in [2] and [3]. The subset $\Sigma_{A,n}(H) = \Sigma_{A,n}$ of elements of the form $y = vs$ with s in the alternating group A_n is a

Received May 22, 1961.