THE ABSTRACT THEOREM OF CAUCHY-WEIL

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1. Let M be a separable, complex-analytic manifold. It is well-known that, if f is a bounded analytic function on M and $p \in M$, then $f(p)$ can be expressed as an integral of the "boundary values" of f. In general the boundary on which the integration is carried out and the boundary values to be integrated are abstract but in special cases a concrete description can be given. Suppose that M is an open subset having compact closure in some larger manifold M' and we consider only analytic functions f on M which have continuous extensions to $\bar{M}$. Then the boundary B is a subset of the topological boundary of M, the boundary values are given by the continuous extension and we may write

$$f(p) = \int_B f(b)d\mu_p(b) \tag{1.1}$$

where μ_p is a measure on B which is independent of f. When M is a region in the plane with rectifiable Γ we have the familiar Cauchy integral formula

$$f(z) = \frac{1}{2\pi i}\int_\Gamma \frac{f(t)dt}{t-z} .$$

Here the integral is expressed with a fixed measure and a kernel function which is analytic in z. In the abstract proof of (1.1) each of the measures μ_p is derived by a separate application of the Hahn-Banach theorem so we cannot directly replace (1.1) with a formula involving an integral kernel depending analytically on p. It is the object of this paper to prove that this is possible.

An explicit formula involving an integral kernel for domains in C^n having sufficiently smooth boundary has been given by Weil [7]; a new proof of this formula in slightly more general circumstances will appear in [3]. Proofs under various circumstances have also been given by Arens [1], Hervé [4] and others. In the present paper we require no regularity on the boundary, but the integral kernel is not given explicitly. Perhaps the most remarkable fact about our result is that in the ordinary complex plane it is new or at least not standard.

2. This section is devoted to incorporating into the literature a proof of the well-known fact that several, apparently different, definitions of an analytic mapping of a complex-analytic manifold into a

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