SOME EXACT SEQUENCES IN COHOMOLOGY THEORY FOR KÄHLER MANIFOLDS

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1. Introduction. In this note some results announced in [1] concerning exact sequences for Kähler manifolds are proved. The main result is that there exists an exact sequence relating the usual bigraded groups of harmonic forms on a compact Kähler manifold K and an imbedded submanifold L with certain mixed relative cohomology groups of (K, L). These mixed cohomology groups have been introduced by Hodge [6]. Using the results of Hodge the exactness of the given sequence is derived in a straightforward manner. H. Guggenheimer considered in [5] such a sequence too. Finally the exact sequence is applied to deduce some results concerning the imbedding of a complex manifold L in a Kähler manifold K, in particular the following statement is proved: if the imbedding $L \subset K$ is homology faithful, then $b_{r,s}(K) = b_{r,s}(L)$ implies $b_{r-k,s-k}(K) = b_{r-k,s-k}(L)$ for $k = 0, 1, 2, \cdots$ ($b_{r,s}$ = rank of the module of harmonic (r, s)-forms). The paper is organized the following way: § 2 gives the necessary notations and a few known results; § 3 contains the main theorem (Theorem 1) the proof of which is given in § 4; some applications follow in §§ 5 and 6, in particular Theorem 2 which implies the foregoing statement on homology faithful imbeddings.

2. Notations and known results.

(a) We use the following notations:

$F^{r,s}$: group of the complex valued $C^\infty(r + s)$-forms of type (r, s) on a complex manifold $M = M^{2n}$.

$F^k = \sum_{r+s=k} F^{r,s}$.

$d = d' + d''$: exterior differentiation operator on M; $d: F^k \to F^{k+1}$, $d': F^{r,s} \to F^{r+1,s}$, $d'': F^{r,s} \to F^{r,s+1}$ (cf [2], [6]).

$*$: $F^{r,s} \to F^{n-s,n-r}$: Hodge—de Rham duality operator which associates to α its (metrically) dual form $*\alpha$, with the help of a Hermitian metric (of class C^∞) on M (cf. [6] and [3]; $** = (-1)^{k(2n-k)} = (-1)^k$: $F^k \to F^k$).

$\delta = -*d* = \delta' + \delta'' = -(*d''* + *d'*)$ ($\delta' = -*d''*$ resp. $\delta'' = -*d'*$ is an operator of type $(-1, 0)$ resp. $(0, -1)$).

$\nabla = d'd''$, $\nabla^* = \delta''\delta' = (-1)^{k+1}*\nabla*$.

$\Delta = d\delta + \delta d$: Laplace—Beltrami operator.

$\Delta' = d'\delta' + \delta'd'$, $\Delta'' = d''\delta'' + \delta''d''$.

We define $Z_{d'}^{r,s} = \{\alpha \mid \alpha \in F^{r,s}, d'\alpha = 0\}$, similarly $Z_{d''}^{r,s}$, $Z_{\nabla}^{r,s}$, $Z_d^{r,s} = Z_{d',d''}^{r,s}$,

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