A SPECTRAL THEORY FOR A CLASS OF LINEAR OPERATORS

G. K. LEAF

In this paper we introduce a type of spectral theory for bounded operators in a Banach space. We shall focus most of our attention on the continuous spectrum, since the point spectrum, at least when it is isolated, can be handled by using the contour integral techniques developed by F. Riesz, E. R. Lorch, and N. Dunford and discussed in [4, 7].

In 1941, E. R. Lorch [6] treated a class of operators in a reflexive Banach space which are natural generalizations of unitary operators. By using ingenious methods, he was able to find invariant manifolds for these operators and constructed a spectral theory which in many respects is similar to that which is available for unitary operators. More recently, N. Dunford [2, 3] has developed an extensive spectral theory for certan classes of operators in Banach spaces, and related work in this area has been done by F. Wolf [9] and others. However, Dunford's class of spectral operators does not contain the class studied by Lorch.

In this paper we will employ some of Dunford's techniques to obtain results which parallel those of Lorch [6]. In doing so, we are able to handle a larger class of operators than in [6], at least in the case where the spectrum is entirely continuous. Finally, we wish to point out that the results in Section 2 are not best possible.

1. Preliminary remarks. If T is a bounded linear operator in a complex Banach space X, then $R(z; T)$ will denote the resolvent operator $(z - T)^{-1}$ defined for z in the resolvant set of T. When T is understood, the notation $R(z)$ will be used in place of $R(z; T)$. For any two points z_1 and z_2 in the resolvant set, R satisfies the following relations:

(i) $R(z_1) - R(z_2) = (z_2 - z_1)R(z_1)R(z_2)$, and

(ii) $R(z_1)R(z_2) = R(z_2)R(z_1)$.

One consequence of the above relations is the analyticity of the vector-valued function $R(z)x$ on the resolvant set $\rho(T)$ for each vector x in the space X. Since $R(z)x$ is a vector-valued analytic function on $\rho(T)$, it is natural to speak of analytic extensions of $R(z)x$. The

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