# SCATTERING FOR NON-LINEAR WAVE EQUATIONS

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**Introduction.** Let $H$ be a Hilbert space, $A$ a positive densely defined self-adjoint linear operator in $H$ (i.e. $A \geqq c_0 I > 0$), $M_t(u)$ a family of (possibly) non-linear operators with domain and range in $H$ and depending on the real parameter $t$, $-\infty < t < +\infty$.

Consider the generalized "wave equation"

$$\frac{d^2u}{dt^2}(t) + (Au)(t) + M_t(u(t)) = 0 \tag{1}$$

where solutions are functions $u(t)$ from the real line $E^1$ to $H$. The equation (1) may obviously be regarded as a perturbation of the simpler equation

$$\frac{d^2u}{dt^2} + Au = 0. \tag{2}$$

The scattering problem for the perturbed equation (1) consists of the following:

(I). *Let $u_0(t)$ be any solution of equation* (2). *For any real number $s$, prove the existence of a solution $u_s(t)$ of the perturbed equation* (1) *such that*

$$u_s(s) = u_0(s)\,; \left(\frac{du_s}{dt}\right)(s) = \left(\frac{du_0}{dt}\right)(s)\,. \tag{3}$$

(II). *Show that as $s \to \pm\infty$, $u_s(t)$ converges in some suitable sense to solutions $u_{\pm\infty}(t)$ of equation* (2). *In this case, we define $W_-(u_0) = u_{-\infty}(t)\,;\ W_+(u_0) = u_{+\infty}(t)$.*

(III). *Study the properties of the operators $W_-$ and $W_+$ defined in* (II), *show the existence of $W_+^{-1}W_- = S$, and study the properties of the scattering operator $S$.*

In a preceding paper [5], the second-named author has solved the scattering problem for equation (1) under the hypothesis that there exist a summable function $\theta(t)$ on $E^1$ such that

$$\| A^{1/2}[M_t(u) - M_t(v)] \| \leqq \theta(t) \| Au - Av \| \tag{4}$$

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