

FREE EXTENSIONS OF BOOLEAN ALGEBRAS

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Introduction. This paper is concerned with the problem of imbedding a Boolean algebra B into an α -complete Boolean algebra B^* in such a way that certain homomorphisms of B can be extended to B^* . We investigate two such imbeddings which arose naturally from the consideration of the work of Rieger and Sikorski in [5] and [7]. In [5] Rieger proved the existence of a certain class of free Boolean algebras and investigated their representability by α -fields of sets. Rieger's theorem on the existence of "the free α -complete Boolean algebra on m generators" is equivalent to the following statement: Every free Boolean algebra B can be imbedded in an α -complete Boolean algebra B^* such that every homomorphism of B into an α -complete Boolean algebra C can be extended to an α -homomorphism of B^* into C . The question now arises: Does this result hold for an arbitrary Boolean algebra B which is not necessarily free? If such an imbedding exists, we call B^* the *free α -extension of B* .

In [7], Sikorski gave a characterization of all the σ -regular extensions of a Boolean algebra B . To obtain this characterization, he first proved that B can be imbedded as a σ -regular subalgebra of a σ -complete Boolean algebra B^* such that every σ -homomorphism of B into a σ -complete Boolean algebra C can be extended to a σ -homomorphism of B^* into C . We call B^* the *free σ -regular extension of B* .

In § 2 of this paper we prove that the free α -extension B_α of B exists uniquely for every Boolean algebra B and every infinite cardinal number α . In § 3 we investigate the representability of B_α by an α -field of sets. We first prove that B_α is isomorphic to an α -field of sets if and only if it is α -representable. A corollary to this result is that the free σ -extension B_σ of an arbitrary Boolean algebra B is isomorphic to a σ -field of sets. The problem of characterizing those Boolean algebras B for which B_α is α -representable for $\alpha \geq 2^{\aleph_0}$ is also discussed. In § 4 we investigate the α -regular extensions of Boolean algebras for an arbitrary cardinal number α . Sikorski's results on the σ -regular extensions depend on the Loomis-Sikorski theorem which does not hold for uncountable cardinal numbers. We use our results on the free α -extension B_α of B to prove the existence of the free α -regular extension and to give a characterization of the α -regular

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