FINITE NETS, II. UNIQUENESS AND IMBEDDING

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1. Introduction. In discussing the present paper we have a choice of three languages: (a) the language of orthogonal latin squares; (b) the language of incomplete block designs, as used in connection with design of experiments; and (c) the geometric language of nets. As far as proofs are concerned, either (b) or (c) affords a useful symmetry which is missing in (a); it is merely a matter of taste that we choose (c). Here let us begin with (a).

Let C be a collection of t mutually orthogonal latin squares of side n. We assume $n > 1, t \geqq 1$. The inequality $t \leqq n - 1$ necessarily holds; if $t = n - 1$, C is said to be *complete*. As is well known, a complete set of orthogonal latin squares of side n determines and is determined by an affine plane of order n. We define the *degree*, k, and *deficiency*, d, of C by

$$k = t + 2 , \qquad d = n - 1 - t , \tag{1.1}$$

so that

$$k + d = n + 1 . \tag{1.2}$$

Here k is, in language (b), the number of *constraints:* one constraint for the rows of the squares, one for the columns, and one for each of the t squares. On the other hand, if C can be enlarged to a complete set, C', of $n - 1$ mutually orthogonal latin squares, then d is the number of squares in C' which are not in C; or the number of constraints missing in C. In language (c) we may describe C as a net N of order n, degree k, deficiency d. For an example of such a net N, we may begin with an affine plane π of order n—with its n^2 points and $n + 1$ parallel classes of lines, n lines per class—and retain the points but delete some d parallel classes.

Before discussing the results of the paper, it will be convenient to define two polynomials $p(x)$, $q(x)$:

$$p(x) = \tfrac{1}{2}x^4 + x^3 + x^2 + \tfrac{3}{2}x , \tag{1.3}$$

$$q(x) = 2x^3 - x^2 - x + 1 , \tag{1.4}$$

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