# ORDERED COMPLETELY REGULAR SEMIGROUPS

TÔRU SAITÔ

**Introduction.** In the previous papers [7], [8], we studied some special types of ordered semigroups in our general sense (cf. § 1). In the continuation of our study, in this note we characterize ordered completely regular semigroups.

In the algebraic theory completely regular semigroups were first studied systematically by Clifford [1]. He characterized these semigroups when the idempotents of the semigroup commute. Recently Fantham [4] generalized this result and characterized completely regular semigroups when the idempotents of the semigroup constitute a subsemigroup.

Ordered completely regular semigroups are shown, in this note, to be semigroups, in which the idempotents constitute subsemigroups. However, it can be shown that the abstract semigroup of an ordered completely regular semigroup is simpler than the semigroups of Fantham's type. Indeed, our main theorem (Theorem 6) asserts that an ordered completely regular semigroup is characterized by the ordered idempotent semigroup constituted by all the idempotents of the semigroup, the ordered groups corresponding to all the elements of the associated semilattice, and the mappings between these ordered groups corresponding to all comparable pairs of elements of the associated semilattice which satisfy certain conditions.

We remark, we characterize, in § 3, ordered completely simple semigroups without zero. This characterization seems to be interesting by itself, by virtue of the importance of completely simple semigroups without zero.

**1. Preliminaries.** A semigroup $S$ is called *completely regular*, if, for every element $a \in S$, there exists $x \in S$ such that

$$axa = a \quad \text{and} \quad ax = xa$$

(Lyapin [5]). Clifford called such a semigroup a semigroup admitting relative inverses and gave many interesting results of this semigroup. Here we mention some of them without proofs.

LEMMA 1 (Clifford [1], Theorem 1). *A semigroup is completely regular if and only if it is the set-union of mutually disjoint groups.*

The adjective 'mutually disjoint' in the above Lemma 1 can be

Received March 27, 1963.