THE SIMPLE CONNECTIVITY OF THE SUM OF TWO DISKS

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1. Introduction. The following question was called to the author's attention several years ago by Eldon Dyer.

Question. Is the sum of two disks simply connected if their intersection is connected?

Later, the author saw a communication in which an erroneous proof was given that Example 1 of the present paper is not simply connected. We show in § 2 that Example 1 is simply connected. However, we give some examples (Examples 2, 3, 4) in §§ 3, 4, 5 that are not simply connected.

A topological characterization is given in § 4 of intersections that will prevent closed curves which finitely oscillate between two disks from being shrunk. If the intersection is snake-like or arcwise connected, such finitely oscillating curves can always be shrunk but there are examples in which infinitely oscillating curves cannot. It is the topology of the intersection which prevents the sum of two disks from being simply connected rather than the embeddings of the intersection in the disks as shown in §§ 4 and 5. In fact, as pointed out in § 6, much of what we have learned about the sums of disks applies to the sums of continuous curves.

We use Example 1 in § 7 to construct a peculiar group and show that a certain relation kills it.

All sets treated in this paper are metric.

Let I^n denote an *n-cell* and *Bd* I^n its *boundary*. A set A is *n-connected* if each map (continuous transformation) f of *Bd* I^{n+1} into A can be extended to map I^{n+1} into A. We say that $f(Bd\ I^{n+1})$ *can be shrunk to a point* if the map can be extended. A set is called an *ε-set* if its diameter is less than or equal to ε. A set A is *n-ULC* if for each $\varepsilon > 0$ there is a $\delta > 0$ such that each map of *Bd* I^{n+1} onto a δ-subset of A can be shrunk to a point on an ε-subset. A compact continuum is called a *continuous curve* if it is 0-*ULC*. A set is *simply connected* if it is 1-connected. It is *uniformly locally simply connected* if it is 1-*ULC*. We shall not treat higher types of connectivity in this paper.

We find it convenient to consider an abstract disk D rather than the square I^2. A map of *Bd* D is a *closed curve*. If h is a homeomorphism, $h(BdD)$ is a *simple closed curve*.

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