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ON AN EXTENSION OF THE PICARD-VESSIOT THEORY

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In previous papers, the author has extended the Galois correspondences between differential Picard-Vessiot extensions and algebraic matrix groups to Picard-Vessiot extensions of a wider class of fields with operators, the so-called M-fields. In this paper, M-field extensions which generalize extensions by integrals and by exponentials of integrals are studied.

These fields are found to be simple field extensions and their structure in the case that the extension is algebraic is investigated. Under suitable restrictions on the fields of constants, the M-Galois groups of these fields are shown to be commutative. Criteria are established for such solution fields to be P–V extensions of M-fields of difference and differential type. An extension obtained by a finite sequence of algebraic extensions, extensions by integrals, and extensions by exponentials of integrals, is called a generalized Liouville extension. It is demonstrated that if the connected component of the identity element in the M-Galois group of a regular P–V extension is a solvable group, then the P–V extension is a generalized Liouville extension, and if a P–V extension is contained in a generalized Liouville extension then the connected component of the identity element in the M-Galois group of the P–V extension is solvable.

1. Terminology and notation are briefly considered in § 2, and a preliminary result on the constants of an algebraic M-extension of an M-field is obtained. The structure of solution fields analogous to extensions by integrals and criteria for the existence of P–V extensions of this type are determined in § 3, and a similar study of solution fields analogous to extensions by exponentials of integrals is made in § 4. In § 5, generalized Liouville extensions are defined, and solvability of the Galois group of a P–V extension is interpreted in terms of imbedding the extension in a generalized Liouville extension.

2. ***M*-rings.** The terminology and notaion of this paper are the same as in [6] and [7]. Let C be an associative, commutative coalgebra with identity over a ring W, which is freely generated as a W-module by a set M. If $w \rightarrow \bar{w}$ is a homomorphism of W into a ring S, let C^s be the S-module obtained from the W-module C by

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