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TOPOLOGICAL SEMILATTICES ON THE TWO-CELL

To Professor A. D. Wallace on his 60th birthday

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Topological lattices on the n-cell have been studied by L.W. Anderson, A.D. Wallace, A.L. Shields, and L.E. Ward, Jr. In particular, these authors have papers setting forth conditions under which a topological lattice on the two-cell is topologically isomorphic to the product lattice $I \times I$.

The primary purpose of this paper is the investigation of topological semilattices (commutative, idempotent topological semigroups) on the two-cell which retain the lattice like property that for each element x, $\{y: x \leqq y\}$ is a connected set. Specifically, it is shown that any such entity is the continuous homomorphic image of one of a fixed pair of semilattices on the two-cell, where the choice of domain depends on the location of the zero element.

It is also proved that a TSL on the two-cell has an identity (a unique maximal element) and $\{y: x \leqq y\}$ connected for each element x if and only if it is the continuous homomorphic image of $I \times I$. Also, if $\{y: x \leqq y\}$ is connected for each element x, then S, a TSL on the two-cell, is generated by its boundary B in the sense that $B^2 = S$.

Semilattices on the n-cell are also discussed. Let S be such an object with boundary B. It is proved that if x is a maximal element of S, then $x \in B$. If S has an identity, 1, and T is a continuum chain from 1 to 0, then $S = BT$.

Finally, let S be a continuum TSL with 1 and let A be the subset defined by $x \in A$ if and only if $\{y: x \leqq y\}$ is connected. Then (1) $x \in A$ if and only if there is a continuum chain from 1 to x; and(2) A is a nondegenerate continuum sub-TSL of S.

Topological lattices on the n-cell have been studied in [1], [6], and in [8]. In particular, these papers set forth conditions under which a topological lattice on the two-cell is iseomorphic (topologically isomorphic) to the product lattice $I \times I$.

The primary purpose of this paper is the investigation of topological semilattices (commutative, idempotent topological semigroups) on the two-cell which retain the lattice-like property that for each element x, $M(x)$ is a connected set (see below). Specifically, we show that any such entity is the continuous homomorphic image of one of a fixed pair of semilattices, where the choice of domain depends upon the location of the zero.

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