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SOME GENERAL PROPERTIES OF MULTI-VALUED FUNCTIONS

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The object is to determine what theorems for single-valued functions can be extended to which class of multi-valued functions. It is shown that an arc cannot be mapped onto a circle by a continuous, monotone multi-valued function when the image of each point is an arc. On the other hand, the arc can be mapped onto a nonlocally connected space by a monotone, continuous function such that the image of each point is an arc. Characterizations of nonalternating functions analogous to the results in the single-valued theory are obtained, and it is shown that an nonalternating semi-single-valued continuous function on a dendrite is monotone. An analog of the monotone light factorization theorem is obtained for semi-single-valued continuous functions.

Some other results are: an open continuous function with finite images maps a regular curve onto a regular curve, and a continuous function with finite images maps a locally connected, compact space onto a locally connected compact space.

A number of definitions for continuity have been proposed for multi-valued or set-valued functions, and Wayman Strother studied the problem of continuity extensively [10, 11, 12]. Also Choquet [2] has studied upper and lower semi-continuous functions. Further, Berge, unlike most authors, allows functions to be multi-valued in [1]. However, much of the work that has been done on set-valued functions has been devoted to the discovery of fixed point theorems ([3], [7] through [9], [11], [13], and [15] through [17]). The purpose of this paper is to investigate properties of multi-valued functions which are similar to the properties of single-valued functions studied in G. T. Whyburn's book, Analytic Topology, [18].

We shall use the following topology on the set of closed subsets of a space Y. Let

$$S(Y) = \{E \subset Y : E \text{ is closed and nonempty}\}.$$

Let $S(Y)$ have the topology used by Michael [6]; i.e., if $V_1, \cdots, V_n$ are open subsets of Y, then the collection $\langle V_1, \cdots, V_n \rangle = \{E \in S(Y) : E \cap V_i \neq \phi$ for all i, and$E \subset \bigcup_{i=1}^n V_i\}$ is a basis for the open sets of $S(Y)$. We shall call this topology the finite topology. This is equivalent to

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