PACIFIC JOURNAL OF MATHEMATICS
Vol. 15, No. 2, 1965

A MORE GENERAL PROPERTY THAN DOMINATION FOR SETS OF PROBABILITY MEASURES

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In posing a statistical problem one specifies a set X, a σ-field S of subsets of X, and a collection M of probability measures on (X, S). It is often convenient to impose some condition on M in order to avoid measure theoretic difficulties and the condition most often used is domination, i.e., the existence of a probability measure with respect to which each of the measures in M is absolutely continuous. In this paper we introduce a more general condition, which we call compactness, implying the existence of a best sufficient subfield and of certain estimates. It is also possible to characterize, under this condition, those functions on M admitting unbiased estimates of certain types.

The increased generality thus afforded should be useful in dealing with certain problems in stochastic estimation where M is not known a priori to be dominated. In any case it is hoped that the present exposition, which leans heavily on some of the more elementary parts of functional analysis, will appeal to those who are oriented toward that subject.

1. The compactness condition.

We will assume throughout this paper that the field S is closed with respect to M, that is that S contains every set whose outer measure is 0 for each μ in M. Such sets will be referred to hereafter as M-null sets.

For each μ in M, S-measurable f and real number p with $1 \leqq p < \infty$ we will write $||f||_{p,\mu}$ for the (finite or infinite) number $\left[\int |f|^p \, d\mu\right]^{1/p}$ and $||f||_{\infty,\mu}$ for the μ-essential supremum of $|f|$. For all $p \geqq 1$ we define

$$||f||_{p,M} = \sup_{\mu \in M} ||f||_{p,\mu}$$

and write $E_p(X, S, M)$ for the set of f with $||f||_{p,M} < \infty$. In what follows, whenever no confusion can result, we will write E_p for $E_p(X, S, M)$ and $||f||_p$ for $||f||_{p,M}$. We will also use the same symbol for a measurable function as for its equivalence class in $E_p(X, S, M)$.

LEMMA 1.1. *E_p with $||f||_p$ as norm is a Banach space.*

Proof. Only the completeness of E_p needs to be proved. If (f_n)

Received July 17, 1963, and in revised form March 31, 1964.