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SAMPLE FUNCTIONS OF CERTAIN DIFFERENTIAL PROCESSES ON SYMMETRIC SPACES

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In a recent paper, we have proved a formula characterizing the abstract Fourier-Stieltjes transform of an isotropic infinitely divisible probability measures on a symmetric space. The formula is the full analogue of the classical Lévy-Khinchine formula for the Fourier-Stieltjes transform of infinitely divisible probability measures on the real line.

Now, just as in the case of the line, an isotropic, infinitely divisible probability measure on a symmetric space gives rise in a natural way to a continuous one parameter convolution semigroup of such measures; and thence to a stochastic process with stationary independent "increments". It is the purpose of this paper to construct the sample functions of such a process. We shall exhibit the sample functions of such a process as limits with probability one (*uniformly* on compact subsets of the parameter set) of sequences of continuous Brownian trajectories interlaced with finitely many isotropic Poissonian jumps.

Our construction brings out clearly the significance of the Lévy measure of the process as a measure of the expected number of jumps of the path having a given size and occurring in unit time. (See details below.) It also follows from our construction that the sample function of these processes can be assumed to have only discontinuities of the first kind. This fact, however, was known and indeed a more general result of this kind was proved in [13] by J. Woll. Thus the main new results of this paper must be considered to be the actual construction of the sample paths, and the geometric information that it gives about the process.

Our results are inspired by the work of Itô [8]. Itô considers such processes on the line. However, the noncommutativity of the groups that concern us and the nonlinear nature of our spaces force us to use techniques quite different from his. Our methods are of independent interest and indeed they can be utilized to construct a theory of "addition" of isotropic random variables taking values in symmetric spaces.

We consider in this paper only the case of a noncompact symmetric space. Surprisingly enough, the compact case is somewhat more messy in technique, due to the fact that in compact symmetric spaces the conjugate locus of a given point interferes with a routine

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