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ON THE CONTINUOUS IMAGE OF A SINGULAR CHAIN COMPLEX

GLEN E. BREDON

A continuous surjection $\pi : X \to Y$ between topological spaces is called "ductile" if, for each $y \in Y$ and neighborhood U of y there is a neighborhood V of y which contracts to y through U in such a way that this contraction can be covered by a homotopy of $\pi^{-1}(V)$. It is shown, in this note, that if $\pi : X \to Y$ is ductile and Y is paracompact then the inclusion of the image $\pi_* C_*(X)$ of the singular chain complex of X in the singular chain complex $C_*(Y)$ of Y induces an isomorphism in homology. Thus $H_*(Y)$ can be computed from those singular simplices of Y which are images of singular simplices of X.

This result does not hold, in general, when π is not ductile. This question was brought to our attention (for a specific case) by Klingenberg who plans to use our result in a study of geodesics on a Riemannian manifold. We shall now rephrase the condition that a map be ductile in a more convenient language.

Let $\mathscr{M}$ be the category whose objects are surjective maps $\pi : X \to Y$ between topological spaces and whose morphisms are commutative diagrams

$$\begin{array}{ccc} X & \longrightarrow & X' \\ {\scriptstyle\pi}\big\downarrow & & \big\downarrow{\scriptstyle\pi'} \\ Y & \longrightarrow & Y' \end{array}$$

of continuous maps (where $\pi, \pi' \in \mathscr{M}$). This contains an analogue of homotopy, that is a commutative diagram

$$\begin{array}{ccc} X \times I & \longrightarrow & X' \\ \big\downarrow{\scriptstyle\pi \times 1} & & \big\downarrow{\scriptstyle\pi'} \\ Y \times I & \longrightarrow & Y' \end{array}$$

For $\pi : X \to Y$ and $A \subset Y$ we let π_A denote the restriction $\pi^{-1}(A) \to A$ of π.

We will say that $\pi : X \to Y$ (in $\mathscr{M}$) is *ductile* if, for each point $y \in Y$ and neighborhood U of y, there is a neighborhood V of y with $V \subset U$ such that the inclusion $\pi_V \to \pi_U$ is homotopic (in $\mathscr{M}$) to a map into $\pi_{\{y\}}$. (Thus V contracts, through U, to $\{y\}$ and this contraction is covered by a homotopy of $\pi^{-1}(V)$.)

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