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QUASI-ISOMORPHISM FOR INFINITE ABELIAN p-GROUPS

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This paper is concerned with the investigation of two closely related questions. The first question is: What relationships exist between G and nG where G is an Abelian group and n is a positive integer?

It is shown that if G' and H' are Abelian groups, n is a positive integer and $nG' \cong nH'$, then $G \cong H$ where $G' = S \oplus G$ and $H' = T \oplus H$ such that S and T are maximal n-bounded summands of G' and H', respectively. A corollary of this is: Every automorphism of nG can be extended to an automorphism of G.

We define two primary Abelian groups G and H to be quasi-isomorphic if and only if there exists positive integers m and n and subgroups S and T of G and H, respectively, such that $p^nG \subset S$, $p^mH \subset T$ and $S \cong T$, the second question is: What does quasi-isomorphism have to say about primary Abelian groups? It is shown that if two Abelian p-groups G and H are quasi-isomorphic then G is a direct sum of cyclic groups if and only if H is a direct sum of cyclic groups, G is closed if and only if H is closed, and G is a Σ-group if and only if H is a Σ-group.

In this paper the word "group" will mean "Abelian group," and we shall use the notation in [5] except that a direct sum of groups A and B will be denoted by $A \oplus B$. Also if $a \in A$ then $H_A^p(a)$ will denote the p-height of a in A. (If it is clear which group or which prime is referred to then either sub- or super-script may be dropped or both.)

At a symposium on Abelian groups held at New Mexico State University, L. Fuchs asked the question: What does quasi-isomorphism (see Definition 3.2) have to say about primary Abelian groups? A question posed by John M. Irwin that arises in the investigation of this question is: What relationships exist between G and nG where G is an Abelian groups and n is a positive integer? The purpose of this paper is to investigate these two questions.

First, we will begin by considering to what extent nG determines G where G is a group and n is a positive integer. It will be shown that if G' and H' are groups, n is a positive integer, and $nG' \cong nH'$, then $G \cong H$ where $G' = S \oplus G$ and $H' = T \oplus H$ such that S and T

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