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SOME NEW RESULTS ON SIMPLE ALGEBRAS

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This paper deals with the problem of proving that a simple algebra (finite dimensional) has an identity element. The main result is contained in the following theorem. Let A be a simple algebra (char. $\neq 2$) in which $(x, x, x) = 0$ and $x^3 \cdot x = x^2 \cdot x^2$. If M is a subset of A such that $(A, M, A) = 0$ and $(M, A, A) \cup (M, A) \cup (A, A, M) \subseteqq M$, then $M = 0$ or there is an identity element in A. This result is then used to prove the three following corollaries (char. $\neq 2$): (1) A simple power associative algebra with all commutators in the nucleus has an identity; (2) A simple power associative algebra with all associators in the middle center has an identity; (3) A simple antiflexible algebra in which $(x, x, x) = 0$ and A^+ is not nil has an identity.

For convenience in terminology, we define an algebra as a finite dimensional vector space on which a multiplication is defined that satisfies both distributive laws. An algebra is nilpotent if there is an integer k such that any product of k elements, no matter how associated, is zero. An element x in an algebra is nilpotent if the subalgebra generated by x is nilpotent. An algebra is nil if it consists entirely of nilpotent elements. A simple algebra is an algebra without proper ideals that is not nil. For char. $\neq 2$, define $x \cdot y = 1/2(xy + yx)$. The algebra A^+ is defined to be the vector space A with miltiplication $x \cdot y$. In addition, define the commutator $(x, y) = xy - yx$ and the associator $(x, y, z) = (xy)z - x(yz)$. Using techniques similar to [3] we will prove the following theorem.

THEOREM 1. *If B is a subspace of an algebra A, there cannot be a nil subset M proper in B with*:

(a) $B = M + MB$

(b) *M a subalgebra*

(c) $(M, M, B) = 0$.

Define $x^1 = x$ and for $k > 1$ define $x^k = x^{k-1}x$. Using this theorem, the following theorem is proved.

THEOREM 2. *Let A be a simple algebra* (char. $\neq 2$) *in which*

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