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EXISTENCE OF HALF-TRAJECTORIES IN PRESCRIBED REGIONS AND ASYMPTOTIC ORBITAL STABILITY

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A theorem is proved concerning the existence of a half-trajectory in the neighborhood of a semi-invariant set of a general dynamical system. A corollary of this theorem strengthens a result of P. Mendelson. The theorem is further used to obtain a necessary and sufficient condition for a compact positively invariant set to be positively asymptotically orbitally stable, and the condition is compared with another one due to S. Lefschetz.

Mendelson [3] applied a topological method due to Wazewski to obtain a sufficient condition for a neighborhood of a rest point of an autonomous system of differential equations to contain a half trajectory other than the rest point. The condition is that all points of egress be points of strict egress. Actually, as we show in § 2, this condition is redundant. Any neighborhood of a rest point of a dynamical system defined on an open set in E^n contains a half trajectory other than the rest point (possibly another rest point).

The purpose of this paper is twofold. First, in § 2 we prove a theorem on the existence of a half trajectory in the neighborhood of a semi-invariant set of a general dynamical system and from it deduce three corollaries. Corollary 2 is a generalization of Mendelson's theorem ([3], p. 221). Corollary 3 gives a sufficient condition for a neighborhood of a rest point to contain a half trajectory which is not a rest point, and it is shown by example that, in a sense, the condition is the best possible. Second, in § 3 Theorem 1 is used to obtain a necessary and sufficient condition for a compact positively invariant set to be positively asymptotically orbitally stable. This condition is compared with another one due to Lefschetz [2].

An autonomous system of differential equations fails to generate a dynamical system in the sense of Nemytskii-Stepanov [5, part II] if the solutions do not exist for all values of the independent variable. However, although the present discussion pertains to a dynamical system, with minor modifications the results in § 2 can be extended to autonomous systems of differential equations whose right hand sides

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