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ON THE MULTIPLICATIVE EXTENSION PROPERTY

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A subspace M of a Banach algebra B is said to have the multiplicative extension property (abbr. m.e.p.) if whenever L is a linear functional on M of norm not greater than one, L is the restriction to M of a multiplicative linear functional on B. This property is considered in two settings—the measure algebra $M(G)$ of a suitable group, and the disc algebra $A(D)$ of functions analytic in the unit disc with continuous boundary values. The following theorems are proved.

THEOREM 2. **If Q is a compact subset of G such that $M_c(Q)$ has the m.e.p., then (i) for every nonzero $t \in G$, the set $Q \cap (Q - t)$ has μ-measure zero for every continuous measure μ on G, and (ii) $m(Q) = 0$, where m is the Haar measure for G.**

THEOREM 3. **Suppose G contains an independent Cantor set. Then there exists a compact subset Q of G such that for infinitely many $t \neq 0$, $Q \cap (Q - t)$ is countably infinite, and $M_c(Q)$ has the m.e.p.**

THEOREM 4. **There exist infinite dimensional subspaces of $A(D)$ with the m.e.p.**

These last two theorems are proved by constructing examples using a special decomposition of the Cantor set. This decomposition is presented in a separate section to simplify notation.

The multiplicative extension property was formulated by Hewitt in [1] after Hewitt and Kakutani [2] had given examples of such subspaces of the measure algebra of certain locally compact groups. In [1] Hewitt poses the problem of characterizing the subspaces with the m.e.p. in a general Banach algebra, and points out that the question was open even for the algebra $C(X)$ of all continuous complex valued functions on a compact Hausdorff space X. Later Phelps [4], who calls the m.e.p. "property (H)", announced such a characterization for $C(X)$. Phelps has shown that a closed subspace A of $C(X)$ has the m.e.p. if and only if X is homeomorphic to a symmetric compact convex subset of a locally convex space and A is the space of linear functions on X. If B is any commutative Banach algebra with unit and maximal ideal space X, and if M is a closed subspace of B with the m.e.p., then the Gelfand transform is an isometric isomorphism of M onto a closed subspace of $C(X)$ which has the m.e.p. Thus Phelps' result characterizes arbitrary subspaces with the m.e.p., in a sense. However, this result gives no information on whether a given algebra has subspaces

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