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A CONVEXITY PROPERTY

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There exist a variety of conditions yielding convexity of a set, dependent upon the nature of the underlying space. It is the purpose here to define a particular restriction involving n-tuples (the n-isosceles property) on subsets of a straight line space and study the effect of this restriction in establishing convexity. By a straight line space is meant a finitely compact, convex, externally convex metric space in which the linearity of two triples of a quadruple implies the linearity of the remaining two. The principal theorem states that the n-isosceles property is a sufficient condition for a closed and arcwise connected subset of a straight line space to be convex if and only if n is two or three.

In such a space S we use two of the definitions stated by Marr and Stamey (4).

DEFINITION 1. If p, q, r are distinct points of S such that at least two of the distances pq, pr, qr are equal, then the points p, q, r are said to form an isosceles triple in S.

DEFINITION 2. A subset M of S is said to have the double-isosceles three-point property if two connecting segments of each of its isosceles triples belong to M.

A proof of (2) together with (4) shows that if M is a closed connected subset of S and possesses the double isosceles property, then M is convex.

DEFINITION 3. A subset M of S is said to have the n-isosceles property ($n \geqq 2$) provided for every $(n+1)$-tuple $p_1, p_2, \cdots, p_{n+1}$ of distinct points of M such that $p_i p_{i+1} = p_{i+1} p_{i+2}$, $i = 1, 2, \cdots, n-1$, at least n of the connecting segments lie in M.

A comparison of the double isosceles property and n-isosceles property shows that in S the two are equivalent for $n = 2$. For n greater than 2, the double isosceles property clearly implies the n-isosceles property but it is not immediately evident whether the two are equivalent. The question may be raised concerning the conditions under which the n-isosceles property is sufficient to replace the double-isosceles property in the above-mentioned theorem yielding convexity.