PACIFIC JOURNAL OF MATHEMATICS
Vol. 18, No. 3, 1966

# HERMITIAN AND ANTI-HERMITIAN PROPERTIES OF GREEN'S MATRICES

E. A. CODDINGTON AND A. ZETTL

**In this paper hermitian and anti-hermitian properties of the components of Green's matrices of related boundary value problems are studied. Necessary and sufficient conditions, depending only on the matrices defining the boundary conditions, for the components of the Green's matrix of one problem to be hermitian or anti-hermitian with respect to certain components of the kernel matrix of a related problem, are found. It is also shown-for a wide class of problems-that some components of these Green's matrices cannot be hermitian (anti-hermitian).**

The techniques used depend on

(i) A construction of Green's matrices due to J. W. Neuberger [2, Theorem B],

(ii) A new vector-matrix formulation of ordinary linear differential equations developed in [3], and

(iii) Chapter 11 of [1].

Denote by $[a, b]$ a finite interval, each of $A, B, P, Q$ a $k \times k$ complex matrix, and $F = (f_{ij})$ a $k \times k$ matrix of continuous complex valued functions on $[a, b]$ such that

$$f_{ij} = 0 \text{ if } i + j \text{ is even.} \tag{1}$$

Let

$$H = (\bar{f}_{k+1-j, k+1-i}), \quad T = ((-1)^i \delta_{i, k+1-j}) .$$

Consider the following vector matrix equations with boundary conditions:

$$Y' = FY, AY(a) + BY(b) = 0, \tag{2}$$

$$X' = HX, PX(a) + QX(b) = 0. \tag{3}$$

Assume that each problem has only the trivial solution. Let $M$ and $N$ be the unique functions such that

$$M(t, u) = I + \int_u^t F(s)M(s, u)ds ,$$

and

$$N(t, u) = I + \int_u^t H(s)N(s, u)ds ,$$