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TAME SUBSETS OF SPHERES IN E^3

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Let F be a closed subset of a 2-sphere S in E^3. We define F to be tame if F lies on some tame 2-sphere in E^3. The sets F and S satisfy Property $(*, F, S)$ provided Bing's Side Approximation Theorem can be applied in such a way that the approximating 2-sphere S' misses F (that is, $S \cap S'$ lies in a finite collection of disjoint small disks in $S - F$). In this paper we show that Property $(*, F, S)$ implies that F is tame by establishing a conjecture made by Gillman. Other properties which are equivalent to Property $(*, F, S)$ are also given.

If $F_1, F_2, \cdots, F_n$ is a finite collection of closed subsets of S such that Property $(*, F_i, S)$ holds for each i, then Property $(*, \sum F_i, S)$ also holds. We use this result to show that if S is locally tame modulo $\sum F_i$, then S is tame.

Bing's Side Approximation Theorem [8, Theorem 16] can be stated as follows:

THEOREM 0. *If S is a 2-sphere in E^3, V is a component of $E^3 - S$, and $\varepsilon > 0$, then there is a polyhedral 2-sphere S' containing a finite collection $D_1, D_2, \cdots, D_n$ of disjoint disks each of diameter less than ε, and there is a finite collection $E_1, E_2, \cdots, E_r$ of disjoint disks on S, each of diameter less than ε, such that*

1. *there is a homeomorphism of S onto S' that moves no point as much as ε,*
2. *$S' - \sum_{i=1}^{n} D_i \subset V$, and*
3. *$S \cap S' \subset \sum_{i=1}^{r} E_i$.*

If F is a closed subset of the 2-sphere S and V is a component of $E^3 - S$, we define *Property* $(*, F, V)$ to mean that Theorem 0 can be applied relative to S and V with the additional requirement that

4. $(\sum E_i) \cap F = \varnothing$.

Property $(*, F, S)$ is satisfied if Property $(*, F, V)$ holds for each component V of $E^3 - S$.

Gillman has already established that an arc A is tame if A lies on a 2-sphere S and Property $(*, A, S)$ is satisfied; however, he comments that the "natural approach" to the problem requires a certain conjecture which he states and does not prove [13, p. 467]. Theorem 3 establishes this conjecture, and Theorem 6 shows that an arbitrary closed set F on S is tame if Property $(*, F, S)$ holds.

Hosay has announced two sufficient conditions for a closed subset