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ON GENERALIZED CAYLEY-DICKSON ALGEBRAS

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Among those algebras whose multiplication does not satisfy the associative law is a particular family of noncommutative Jordan algebras, the generalized Cayley-Dickson algebras. These are certain central simple algebras whose dimensions are all powers of two. Most of this paper is concerned with giving the classification up to isomorphism of those of dimensions 16, 32, and 64 and determining the automorphism groups. In addition to this some generalized Cayley-Dickson division algebras are constructed. Precise criteria for when the 16-dimensional algebras are division algebras are formulated and applied to algebras over some common fields. For higher dimensions no such criteria are given. However, specific examples of division algebras for each dimension 2^t are constructed over power-series fields.

DEFINITIONS. Let us recall the definition of our algebras. Let $\mathfrak{A}$ be any algebra (not necessarily associative) with an involution $a \rightarrow a^*$, that is, a nonsingular linear transformation on $\mathfrak{A}$ such that $(ab)^* = b^*a^*$ and $(a^*)^* = a$. If γ is a nonzero element of the ground field, we define the algebra $\mathfrak{A}\{\gamma\}$ to be the set of pairs (a, b) with a and b in $\mathfrak{A}$ and with addition and scalar multiplication defined in the obvious way. To avoid confusion with bilinear forms which will be appearing let us write u for $(0, 1)$ and $a + bu$ for (a, b). Multiplication in $\mathfrak{A}(\gamma)$ is then defined by

$$(a + bu)(c + du) = (ac + \gamma d^*b) + (da + bc^*)u .$$

The map $a \rightarrow a + 0u$ imbeds $\mathfrak{A}$ isomorphically in $\mathfrak{A}\{\gamma\}$, and $a + bu \rightarrow a^* - bu$ extends the involution to $\mathfrak{A}\{\gamma\}$. If 1 is a unity element of $\mathfrak{A}$, then $1 + 0u$ is a unity of $\mathfrak{A}\{\gamma\}$.

A generalized Cayley-Dickson algebra $\mathfrak{A}_t$ of dimension 2^t is constructed by choosing nonzero elements $\gamma_1, \cdots, \gamma_t$ in the ground field $\mathfrak{F}$. Then we set $\mathfrak{A}_0 = \mathfrak{F}$ (with the trivial involution $a^* = a$) and $\mathfrak{A}_i = \mathfrak{A}_{i-1}\{\gamma_i\} = \mathfrak{A}_{i-1} \oplus \mathfrak{A}_{i-1}u_i$ for $i = 1, 2, \cdots, t$. The norm $n(x) = xx^* = x^*x$ of x in $\mathfrak{A}_t$ is a multiple of the unity element and can be linearized to give a nondegenerate bilinear form on $\mathfrak{A}_t$:

$$(x, y) = \frac{1}{2}[n(x + y) - n(x) - n(y)]$$
$$= \frac{1}{2}(xy^* + yx^*) .$$

We note that if $n(x) \neq 0$, then x has the inverse $n(x)^{-1}x^*$ and