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RELATIVE SATELLITES AND DERIVED FUNCTORS OF FUNCTORS WITH ADDITIVE DOMAIN

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This paper deals with relative satellites and derived functors of functors from an additive category $\mathfrak{A}$ into an Abelian category. The satellites and derived functors are defined by universal properties relative to classes $\mathfrak{S}$ of morphisms of $\mathfrak{A}$ that contain all morphisms whose domain is an initial object of $\mathfrak{A}$, that are closed under multiplication and base-coextension, and whose elements have cokernels. The existence of satellites and derived functors relative to $\mathfrak{S}$ is shown by a method due to D. Buchsbaum without using the existence of either enough $\mathfrak{S}$-injective or $\mathfrak{S}$-projective objects in $\mathfrak{A}$. With the proper notion of $\mathfrak{S}$-exactness in $\mathfrak{A}$ the exactness of the long satellite resp. derived functor sequence is established under quite general assumptions.

In the absolute case where $\mathfrak{A}$ is an Abelian category and $\mathfrak{S}$ is the class of all monomorphisms of $\mathfrak{A}$ the satellites resp. derived functors relative to $\mathfrak{S}$ are the well-known absolute satellites resp. derived functors as defined by H. Cartan and S. Eilenberg [2, Ch. 3 and 5] resp. P. Gabriel [4, Ch. 2]. The existence and exactness theorems 4.3, 4.5, 4.6, 4.7, 4.9, 5.2, 5.5 and 5.6 of this paper especially furnish the corresponding theorems of H. Cartan and S. Eilenberg [2, Ch. 3 and 5], A. Grothendieck [5, Ch. 2], D. Buchsbaum [1], P. Gabriel [4, Ch. 2], and H. Röhrl [9].[1)]

If $\mathfrak{A}$ is Abelian and $\mathfrak{S}$ is the class of morphisms of an injective structure on $\mathfrak{A}$ in the sense of J. Maranda [7] then one recovers the results of [8]. In the preceding example the assumption that $\mathfrak{A}$ is Abelian is not necessary as has been shown by S. Eilenberg and J. C. Moore [3].

The results of this paper apply in particular to the following cases (§ 6)

(I) The category $\mathfrak{A}$ has enough T-$\mathfrak{S}$-injective objects. In the absolute case one obtains an improvement of the results of H. Röhrl [9].

(II) For every object A in $\mathfrak{A}$ the preordered class $\mathfrak{S}(A)$ of all morphisms in $\mathfrak{S}$ with domain A has a largest element.

(III) The category $\mathfrak{A}$ has enough $\mathfrak{S}$-injective objects. This case has been dealt with in [3].

1) After submission of this paper to the editor a paper of F. Ulmer *Satelliten und derivierte Funktoren*, Math. Z. **91** (1966), appeared. Ulmer also uses the method of D. Buchsbaum [1], and obtains, for the absolute case, results similar to ours.