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A NOTE OF DILATIONS IN L^p

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The objects of study in this note are the Lebesgue spaces $L^p(1 < p < \infty)$ on the n-dimensional Euclidean space R^n. We consider a function f in one of the above-mentioned spaces, and derive results about the closure (in the relevant function space) of the set of linear combinations of functions of the form

$$f(a_1x_1 + b_1, \cdots, a_nx_n + b_n)$$

where $a_1, \cdots, a_n, b_1, \cdots, b_n \in R$, and $a_1 \neq 0, \cdots, a_n \neq 0$.

1. Notation and main results. The Haar measure on R^n will be denoted by dx. It will be assumed normalized so that the Fourier inversion formula holds without any multiplicative constants outside the integrals involved.

If $x \in R^n$, and k is an integer such that $1 \leqq k \leqq n$, then x_k will denote the k-th component of x. Multiplication (and of course addition) in R^n is defined component-wise, in the usual manner.

We write $R^* = R^n \backslash \{x: x_k = 0 \text{ for some } k\}$.

Suppose that $1 < p < \infty$. Then q will always be written for the number satisfying

$$\frac{1}{p} + \frac{1}{q} = 1 .$$

For each integer k such that $1 \leqq k \leqq n$, J_k will denote the projection of R^n onto its k-th factor; *i.e.*

$$J_k(x) = x_k \text{ for all } x \in R^n .$$

If f is any function on R^n, and $a \in R^*, b \in R^n$, then f_b^a will denote the function defined by

$$f_b^a(x) = f(ax + b) \text{ for all } x \in R^n .$$

(The map $x \rightarrow ax + b$ is called a dilation of R^n.) Finally, the set S_f is defined by

$$S_f = \{f_b^a: a \in R^*, b \in R^n\} .$$

In what follows, several vector spaces will be considered. If $1 < p < \infty$, $L^p(R^n)$ will denote the usual Lebesgue space. $L^p(R^n)$ will be given the usual norm topology.

If f is an element of $L^p(R^n)$ we shall denote by $\boldsymbol{T}[f]$ the closed vector subspace of $L^p(R^n)$ generated by S_f.