PACIFIC JOURNAL OF MATHEMATICS
Vol. 22, No. 1, 1967

ON EVANS' KERNEL

MITSURU NAKAI

In classical potential theory on the plane, two important kernels are considered: the hyperbolic kernel $\log(|1-\bar{\zeta}z|/|z-\zeta|)$ on $|z|<1$ and the logarithmic kernel $\log(1/|z-\zeta|)$ on $|z|<+\infty$. The former is extended to a general open Riemann surface of positive boundary as the Green's kernel.

The object of this note is to generalize the latter to an arbitrary open Riemann surface of null boundary, which we shall call Evans' kernel. The symmetry (Theorem 1) and the joint continuity (Theorem 2) of Evans' kernel are the main assertions of this note. It is also shown that Evans' kernel is obtained on every compact set in the product space as a uniform limit of Green's kernels of specified subsurfaces less positive constants (Theorem 3).

The hyperbolic and logarithmic kernels are characteristic of hyperbolic and parabolic simply connected Riemann surfaces, respectively. The corresponding rôle is played by the elliptic kernel $\log(1/[z,\zeta])$ for an elliptic simply connected Riemann surface, i.e., a sphere. The generalization of it, which we call Sario's kernel, is shown to be obtained in a natural manner from the Evan's kernel.

Wide applications of Evan's kernel are obviously promised, but we do not discuss these here at all.

1. Positive singularities. Throughout this note, we denote by R an open Riemann surface of null boundary, i.e., $R \in 0_G$ (cf. Ahlfors-Sario [1]). We denote by $\tilde{R}$ the one point compactification of Alexandroff and by ∞ the point at infinity, i.e., $\tilde{R} = R \cup \{\infty\}$ (cf. Kelly [4]).

Let $q \in \tilde{R}$. A *positive singularity* (or more precisely, normalized positive singularity) l_q at q is a positive harmonic function in a punctured open neighborhood $V(l_q) \subset R$ (i.e., $V(l_q) \cup \{q\}$ is an open neighborhood of q in $\tilde{R}$) such that

$$\lim_{p \in V(l_q),\, p \to q} l_q(p) = +\infty \tag{1}$$

and

$$\int_\alpha {}^*dl_q = -2\pi \tag{2}$$

for a (and hence for all) simple analytic curve $\alpha \subset V(l_q)$ which is the boundary of a neighborhood of q and is positively oriented with respect to this neighborhood. Here ${}^*dl_q$ is the conjugate differential of dl_q