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THE DILATATION OF SOME STANDARD MAPPINGS

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It is not unusual to consider on a surface a conformal structure determined by a positive definite quadratic form which may or may not be the official Riemannian metric on the surface. Given a smooth mapping with positive Jacobian between a pair of surfaces each provided with such a conformal structure, we describe in this paper an obvious procedure for computing the dilatation of the mapping. Next, we consider surfaces smoothly immersed in E^3, and mappings (called allowable) for which dilatation is a function of the principal curvatures at corresponding points. Referring to a conformal structure as geometrically significant if determined by a linear combination of the fundamental forms with coefficients which are smooth functions of the principal curvatures, we show (for example) that a mapping which preserves lines of curvature is allowable between any pair of geometrically significant conformal structures if it is allowable between any one pair of geometrically significant conformal structures. Finally, we prove that a complete surface smoothly immersed in E^3 on which $K \leqq 0$ and $H^2 - K \equiv c \neq 0$ is conformally equivalent either to the finite plane or to the once punctured finite plane.

1. Computing dilatation. The material in this section is not in any sense original. (See, for example, [2], p. 103 and 118, a source suggested by the referee.) What we present is the procedure by which dilatation will be computed throughout the remainder of the paper.

Consider a pair of abstract, smooth, oriented surfaces S and $\hat{S}$, and a smooth mapping

$$f: S \rightarrow \hat{S}$$

with positive Jacobian. Next, suppose that an arbitrary pair of smooth positive definite quadratic forms Λ and $\hat{\Lambda}$ are used to determine conformal structures on S and $\hat{S}$, respectively, yielding Riemann surfaces R_Λ and $\hat{R}_{\hat{\Lambda}}$ ([1], p. 26). At each point p of S, the induced mapping

$$f: R_\Lambda \rightarrow \hat{R}_{\hat{\Lambda}}$$

has the dilatation ([1], p. 1)

$$\mathscr{K}_f(\Lambda, \hat{\Lambda})\,|_p = \frac{\max \sqrt{\dfrac{\hat{\Lambda}\,|_{f(p)}}{\Lambda\,|_p}}}{\min \sqrt{\dfrac{\hat{\Lambda}\,|_{f(p)}}{\Lambda\,|_p}}} \tag{1}$$