PACIFIC JOURNAL OF MATHEMATICS
Vol. 22, No. 1, 1967

CLOSED OPERATORS AND THEIR ADJOINTS ASSOCIATED WITH ELLIPTIC DIFFERENTIAL OPERATORS

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We are concerned here with determining some closed operators associated with a given elliptic differential operator A of order $2m$ and some in general nonlocal boundary operators. We seek conditions in particular which guarantee that the result is a normally solvable operator, i.e. with closed graph and closed range in the sense of Visik. We follow basically the method used in Bade and Freeman in the sense that we regard the operator with nonlocal boundary conditions as a perturbation of an operator with boundary conditions defined by a normal set of differential operators $B = \{B_0, \cdots, B_{m-1}\}$ satisfying the condition of Agmon, Douglis and Nirenberg (also Browder, and Schechter). Since the basic a priori estimate valid for such systems essentially says that the resulting operator has closed graph we call such a system (A, B) closable elliptic.

In addition to dealing with higher order elliptic operators and general boundary conditions we also drop the requirement that our region be relatively compact and instead make the weaker requirement that the differential operator in $H^{2m}(\Omega)$ with local boundary conditions yields an operator with closed range. We work here in L^2 only and consider operators defined in $H^{2m}(\Omega)$, in the graph topology associated with the so called maximal operator and in a family of spaces interpolated between these two. Most of our results can be obtained, at least for relatively compact regions, in L^p with $1 < p < \infty$ at the expense of a somewhat more complicated treatment. A particular complication arises from the fact that different interpolation methods which yield the same spaces in L^2 do not in general in $L^p, p \neq 2$.

The paper is divided into eight sections the first five of which are of a preliminary nature and contain results many of which are variants of well known results.

While in the process of writing this paper we were able to see the thesis of R. W. Beals which he kindly sent to us. The two papers are concerned with similar problems but the results cannot be ordered by inclusion.

1. Preliminaries. As usual points in R^n (n-dimensional Euclidean space) are denoted by $x = (x_1, \cdots, x_n)$ and n-dimensional Lebesgue