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ON PRODUCTS OF MAXIMALLY RESOLVABLE SPACES

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A maximally resolvable space is one which can be decomposed into the largest number of "maximally dense" subsets. Answering a previously posed question, we show that an arbitrary product of maximally resolvable spaces is again maximally resolvable, not only with respect to the ordinary product topology, but with respect to other natural topologies as well.

Given a topological space (X, τ) let $\varDelta(X, \tau)$ denote the least among the cardinals of nonvoid τ-open sets. A space (X, τ) is said to be *maximally resolvable* if it has isolated points or X is the union of $\varDelta(X, \tau)$ pairwise disjoint sets, called resolvants, each of which intersects each nonvoid open set in at least $\varDelta(X, \tau)$ points.

In [1] the first author showed among things that locally compact Hausdorff spaces and first countable spaces are always maximally resolvable. Moreover, it was shown that in certain cases the product of maximally resolvable spaces is maximally resolvable. In this paper we settle this question of the maximal-resolvability of products by showing that an arbitrary product of maximally resolvable spaces is maximally resolvable with respect to the ordinary product topology, the generalized product topology, and the box topology. Other results, as well as some interesting unsolved problems regarding products of maximally resolvable spaces, are also presented.

In the sequel we will consider ordinals and cardinals defined so that each ordinal is equal to the set of its predecessors, and a cardinal is an ordinals which is not equipollent with any smaller ordinal. Cardinals will be denoted by the aleph notation or by the symbols $\mathfrak{k}, \mathfrak{m}, \mathfrak{n}$, etc., and ordinals will be denoted by lower case Greek letters α, β, γ, etc.. The cardinal number of a set A will be denoted by $|A|$. A subset B of a topological space X is said to be $\mathfrak{m}$-dense in X if $|B \cap U| \geqq \mathfrak{m}$ for each nonvoid open subset U of X. A subset B of a given set M is said to be *small* (resp. *large*) with respect to M if $|B| < |M|$ (resp. $|M - B| < |M|$). When no confusion is likely, we will denote $\varDelta(X, \tau)$ by $\varDelta(X)$.

Given a Cartesian product $\prod \{X_\alpha : \alpha \in M\}$ of topological spaces X_α, let $\mathscr{U}$ be the collection of all sets of the form $\prod \{U_\alpha : \alpha \in M\}$ where U_α is open in X_α. Let $\mathscr{B}$ be the topology generated by the base $\mathscr{U}$. Let $\mathscr{P}$(resp. $\mathscr{G}$) be the topology generated by the base consisting of all members of $\mathscr{U}$ for which $U_\alpha = X_\alpha$ except for finitely many