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SIMPLIFYING INTERSECTIONS OF DISKS IN BING'S SIDE APPROXIMATION THEOREM

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In Bing's Side Approximation Theorem for 2-spheres in E^3 the disks on the approximating sphere and the disks on the given sphere may intersect in a very complicated manner. It is shown in this paper that the disks may be chosen so that there are the same number of disks on the approximating sphere as on the given sphere and the disks intersect in a one-to-one fashion. Furthermore, the approximating homeomorphism may be chosen so that it maps each disk on the given sphere onto the disk on the approximating sphere which it intersects.

Applications are given to a study of the preservation of tameness of subsets of the boundary of a crumpled cube under re-embeddings of the crumpled cube in E^3.

An ε-mapping of a subset of E^3 into E^3 is a mapping which moves no point a distance as much as ε. An ε-set in E^3 is a set of diameter less than ε. David Gillman [6] and L. D. Loveland [10, 11] have used the following definition in connection with tame subsets of 2-spheres in E^3.

Property $(*, F, V)$. If F is a closed subset of a 2-sphere S in E^3 and V is a component of $E^3 - S$, then $(*, F, V)$ is satisfied if for each $\varepsilon > 0$ there exist an ε-homeomorphism h of S into E^3, a finite collection of mutually exclusive ε-disks $\{D_1, D_2, \cdots, D_r\}$ on $h(s)$, and a finite collection of mutually exclusive ε-disks $\{E_1, E_2, \cdots, E_n\}$ on S such that

(i) $h(s)$ is polyhedral,
(ii) $h(s) - \bigcup \operatorname{Int} D_j \subset V$,
(iii) $h(s) \cap s \subset \bigcup \operatorname{Int} E_i$, and
(iv) $(\bigcup E_i) \cap F = \varnothing$.

Bing's Side Approximation Theorem [3, Th. 16] may then be stated as follows.

THEOREM A (*Bing*). *If S is a 2-sphere in E^3 and V is a component of $E^3 - S$, then $(*, \varnothing, V)$ is satisfied.*

We show in § 6 that if $(*, F, V)$ is satisfied, then the homeomorphism h and the collections of disks may be chosen so that the following additional properties are satisfied.

(v) $r = n$,