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# TENSOR PRODUCTS OF GROUP ALGEBRAS

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**Let $G, H, K$ be locally compact abelian groups where $K$ is noncompact and both the quotient $G/N^G$ where $N^G$ is a compact (normal) subgroup and the quotient $H/N^H$ where $N^H$ is a compact (normal) subgroup. Then in a natural fashion the group algebras $L_1(G)$ and $L_1(H)$ are modules over $L_1(K)$ and**

$$L_1(G) \bigotimes_{L_1(K)} L_1(H) \cong L_1(K) .$$

In [2, 3, 4, 5] there are discussions of tensor products of Banach spaces and Banach algebras over the field $\mathfrak{C}$ of complex numbers and over general Banach algebras. We note the following results to be found in these papers:

(i) If $A, B, C$ are commutative Banach algebras and if $A$ and $B$ are bimodules over $C$ (where $\| ca \| \leqq \| c \| \| a \|$, $\| cb \| \leqq \| c \| \| b \|$, $a \in A$, $b \in B$, $c \in C$) then the space $\mathfrak{M}_D$ of maximal ideals of $D \equiv A \otimes_C B$ may be identified with a subset of $\mathfrak{M}_A \times \mathfrak{M}_B$ as follows:

$$\mathfrak{M}_D = \{(M_A, M_B) : M_A \in \mathfrak{M}_A, M_B \in \mathfrak{M}_B, \mu(M_A) = \nu(M_B) \neq \text{null map}\} .$$

(Here $\mu$ and $\nu$ are continuous mappings of $\mathfrak{M}_A$ and $\mathfrak{M}_B$ into $\mathfrak{M}_C^\circ$ = the maximal ideal space of $C$ with the null map adjoined. These maps are defined as follows: If $a \in A, b \in B, c \in C$ then

$$a^\frown(M_A)c^\frown(\mu(M_A)) = ca^\frown(M_A)$$
$$b^\frown(M_B)c^\frown(\nu(M_B)) = cb^\frown(M_B) .$$

Finally

$$\begin{aligned} c(a \otimes b)^\frown(M_A, M_B) &= c^\frown(\mu(M_A))a^\frown(M_A)b^\frown(M_B) \\ &= c^\frown(\nu(M_B))a^\frown(M_A)b^\frown(M_B) . \end{aligned}$$

[3].)

(ii) If $G, H, K$ are locally compact abelian groups and if $\theta_G : K \rightarrow G$, $\theta_H : K \rightarrow H$ are continuous homomorphisms with closed images, then $L_1(G)$ and $L_1(H)$ are $L_1(K)$-bimodules according to the formulas:

$$ca(\xi) = \int_K a(\xi - \theta_G(\zeta))c(\zeta)d\zeta,\ a \in L_1(G),\ c \in L_1(K) .$$

$$cb(\eta) = \int_K b(\eta - \theta_H(\zeta))c(\zeta)d\zeta,\ b \in L_1(H),\ c \in L_1(K) .$$

Furthermore the mappings $\mu$ and $\nu$ of (i) are simply the dual mappings