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# A NONIMBEDDING THEOREM OF NILPOTENT LIE ALGEBRAS

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**There are many similarities between groups of prime power order and nilpotent Lie algebras. Here we present a nonimbedding theorem in nilpotent Lie algebras which is an analogue of a nonimbedding theorem of Burnside in groups of prime power order.**

Burnside in [2] proved the following two theorems:

THEOREM B1. *A nonabelian group whose center is cyclic cannot be the derived group of a p-group.*

THEOREM B2. *A nonabelian group, the index of whose derived group is $p^2$, cannot be the derived group of a p-group.*

Hobby in [3] proved the following analogues of the theorems of Burnside:

THEOREM H1. *If $H$ is nonabelian group whose center is cyclic, then $H$ cannot be the Frattini subgroup of any p-group.*

THEOREM H2. *A nonabelian group, the index of whose derived group is $p^2$, cannot be the Frattini subgroup of any p-group.*

The purpose of this note is to establish the analogues of the theorems of Burnside in Lie algebras. The main result is the following Theorem 1. The Lie algebras which we consider here are finite dimensional over an arbitrary field $F$. The Frattini subalgebra $\phi(M)$ of a Lie algebra $M$ is defined as the intersection of all maximal subalgebras of $M$. We also show that in a nilpotent Lie algebra $N$, $\phi(N)$ coincides with the derived algebra of $N$. Hence, the analogues of Hobby's theorems in Lie algebras are the same as the analogues of Burnside's theorems in Lie algebras.

THEOREM 1. *A nonabelian Lie algebra $L$ whose center is one dimensional cannot be any $N_i$, $i \geqq 1$, of a nilpotent Lie algebra $N$ where $N = N_0 \supset N_1 \supset N_2 \supset \cdots \supset N_t \supset 0$ is the lower central series of $N$.*

*Proof.* Suppose the contrary, i.e., $L = N_i$ for some $i$, $1 \leqq i < t$,