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ON THE BEHAVIOR OF THE SOLUTION OF THE TELEGRAPHIST'S EQUATION FOR LARGE VELOCITIES

LARRY E. BOBISUD

Consider the solution φ of the inhomogeneous telegraphist's equation with signal velocity $1/\sqrt{\varepsilon}$, where φ satisfies the initial conditions $\varphi(x, 0) = g(x)$, $\varphi_t(x, 0) = h(x)$. Let U be the solution of the parabolic equation that results from the telegraphist's equation when the velocity is set equal to ∞ ($\varepsilon = 0$), and let U satisfy the initial condition $U(x, 0) = g(x)$. Our main result is that, under suitable conditions on the coefficients and data,

$$\int_{-\infty}^{\infty}[\varphi(x, t) - U(x, t)]^2 dx = 0(\sqrt{\varepsilon})$$

uniformly in $0 \leqq t \leqq \bar{t}$ for any finite $\bar{t}$.

We shall investigate the behavior of the solution of the telegraphist's equation

$$\frac{1}{c^2}a'\varphi_{tt} + b'\varphi_t - c'\varphi_{xx} = f' ,$$

where $a' > 0$, $b' > 0$, $c' > 0$, as the velocity c becomes large. For ease of writing, we will treat explicitly only the case of one space dimension, although the method employed will obviously extend to any number of space variables. Since we suppose that c' is never zero, we can write the telegraphist's equation in the form

$$\varepsilon a(x, t)\varphi_{tt} + b(x, t)\varphi_t - \varphi_{xx} = f(x, t) , \tag{1}$$

where $\varepsilon = 1/c^2$ is a small parameter. We shall require that the solution φ satisfy the initial conditions

$$\begin{aligned} \varphi(x, 0) &= g(x) \\ \varphi_t(x, 0) &= h(x) \end{aligned} \tag{2}$$

for each $\varepsilon > 0$. Clearly φ depends on ε; when it is desirable to make this dependence explicit we shall write either φ_ε or $\varphi(x, t; \varepsilon)$, and similarly for other quantities which may depend on ε.

If we set $\varepsilon = 0$ in eq. (1) we get a parabolic equation

$$b(x, t)U_t - U_{xx} = f(x, t) \tag{3}$$

called the *degenerate equation* corresponding to eq. (1). Since this equation is of first order in t, we cannot hope to impose both initial