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IMAGES OF ORDERED COMPACTA ARE LOCALLY PERIPHERALLY METRIC

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In this paper we study the class $\mathfrak{K}$ of Hausdorff compact spaces X which are obtainable as images of ordered compacta K under (continuous) maps $f: K \to X$ onto X. The topology of K is the order topology induced by a total (linear) ordering $<$ on K. We find that X is locally peripherally metric (Theorem 5), i.e., it has a basis of open sets with metrizable frontiers.

In fact, our main result is this stronger statement.

THEOREM 1. *Let X be a continuous image of an ordered compactum K and let G be an open F_σ-set in X. If* Cl G *is connected, then the frontier Fr G is metrizable.*

Theorems 1 and 5 answer in the affirmative two questions raised by the author in [3].

As an immediate consequence, we obtain

COROLLARY 1. *Let X be a continuous image of an ordered compactum K and let G be an open F_σ-set in X. If every point $x \in$* Fr *G has a connected open neighborhood in* Cl G, *then* Fr G *is metrizable.*

Another easy consequence of Theorem 1 is the following theorem of L. B. Treybig [10]:

COROLLARY 2 (*Treybig*). *Let X be a continuous image of an ordered compactum K. If X is connected and separable, then it is metrizable.*

The proof of Theorem 1 given in §5 depends on an apparently new metrization theorem for Hausdorff compact spaces (Theorem 2 of §1), on earlier work of the author on separation properties of images of ordered compacta [3], on the earlier joint work with P. Papić ([5], [6],) and on the following product theorem due to A. J. Ward [13] and L. B. Treybig [9] (see also [3] and [4]).

PRODUCT THEOREM (*Ward, Treybig*). *Let X and Y be infinite compacta such that $X \times Y$ is the image of an ordered compactum. Then both X and Y are metrizable.*

The proof of Theorem 1 does not depend on Corollary 2 and,