PACIFIC JOURNAL OF MATHEMATICS
Vol. 23, No. 3, 1967

# DIOPHANTINE SYSTEMS

A. A. AUCOIN

**We concern ourselves in this paper with integral solutions of three Diophantine systems, generalizations of**

$$x + y + z = u + v + w, xyz = uvw$$

**and of** $xy + xz + yz = uv + uw + vw, xyz = uvw$. **The solutions are given in terms of parameters and are integral for an integral choice of the parameters. Throughout the paper the integer** $n$ **will be greater than 1.**

Heron [3] in the first century B. C. considered the problem of finding two rectangles such that the area of the first is three times the area of the second and the perimeter of the second is three times of perimeter of the first. He also considered a second problem which results in the Diophantine system $x + y = u + v, xy = 4uv$. Planude [3] discussed the system $x + y = u + v, xy = buv$, and Cantor [3] gave general solutions to this problem. Tannery [3] generalized the two problems of Heron. Moessner [7] and [8] gave particular solutions, while Dickson [4] and Gloden [6] gave parametric solutions of the system

$$(1)\qquad \begin{aligned} x + y + z &= u + v + w\ , \\ xyz &= uvw\ , \end{aligned}$$

Bini [1] considered a system equivalent to (1) and Buquet [2] extended this system to $2n$ unknowns.

All of the above systems are special cases of the system

$$(2)\qquad \begin{aligned} A(x, y) &= 0\ , \\ cP(x) &= dP(y)\ , \end{aligned}$$

where $A(\alpha, \beta) = \sum_{i=1}^{n} (a_i\alpha_i - b_i\beta_i)$, $P(\alpha) = \prod_{i=1}^{n} \alpha_i$, $a_i$, $b_i$ are integers, and $c$ and $d$ are nonzero integers. We make the following definitions:

$$A_p(\alpha, \beta) = A(\alpha, \beta) - (a_p\alpha_p - b_p\beta_p),$$

$P_p(\alpha) = P(\alpha)/\alpha_p$, $\pi_1(\alpha, \beta) = cb_pP_p(\alpha) - da_pP_p(\beta)$, $p$ is a fixed integer, $1 \leqq p \leqq n$, and the $\alpha$'s and $\beta$'s are arbitrary integers.

We agree that solutions in which some unknown vanishes, or those for which $a_ix_i = b_iy_i$, $(i = 1, \cdots, n)$ are trivial solutions.

THEOREM. 1. *Any nontrivial integral solution of* (2) *is proportional to a solution given by*