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THE SPECTRAL RADIUS OF AVERAGING OPERATORS

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This paper is concerned with the properties of certain operators which act on rearrangement invariant spaces of functions. The spectral radius of such an operator is determined precisely in terms of the spectral radius of the translation operator E_s. A new inequality is obtained for the norm of the iterates of the averaging operator P in terms of the norm of E_s.

We consider further properties of the operators P_a and Q_a which were exploited in [2] (with a slightly different notation).

If f is a measurable function defined on $R^+ = [0, \infty)$, then $P_a f$ and $Q_a f$ are given by the following:

$$P_a f(t) = \int_0^1 s^{-a} f(st)ds = t^{a-1}\int_0^t s^{-a} f(s)ds \tag{1}$$

$$Q_a f(t) = \int_1^\infty s^{a-1} f(st)ds = t^{-a}\int_t^\infty s^{a-1} f(s)ds \ , \tag{2}$$

whenever the defining integrals exist a.e.

We define the translation operator E_s by

$$E_s f(t) = f(st), \qquad 0 < s < \infty, \qquad t \in R^+ \ . \tag{3}$$

Thus, in some sense which need not be made precise here,

$$P_a = \int_0^1 s^{-a} E_s ds, \quad \text{and} \quad Q_a = \int_1^\infty s^{a-1} E_s ds \ . \tag{4}$$

Now, suppose that X is a rearrangement invariant Banach space as defined in [1]. It was shown in [2] that it is important to know whether or not the operators P_a and Q_a define continuous mappings of X into itself. In terms of the norm of E_s, which we write as $h(s) = \| E_s \|$, a sufficient condition in order that $\| P_a \| < \infty$ is that

$$\int_0^1 s^{-a} h(s)ds < \infty \ , \tag{5}$$

and a bound for the norm is given by

$$\| P_a \| \leqq \int_0^1 s^{-a} h(s)ds \ . \tag{6}$$

A similar expression holds for $\| Q_a \|$. This is a consequence of Theorem 3.1 of [1]. Here, we will always adopt the convention that if P_a is not a bounded operator from all of X into itself then $\| P_a \| = \infty$.