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THE STRUCTURE SPACE OF A COMMUTATIVE LOCALLY M-CONVEX ALGEBRA

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If A is a commutative Banach algebra with identity, then the sets $\mathscr{M}$ (all maximal ideals), $\mathscr{M}_c$ (all closed maximal ideals), $\mathscr{M}_1$ (kernels of nonzero C-valued homomorphisms of A), and $\mathscr{M}_0$ (kernels of nonzero continuous C-valued hommorphisms of A) coincide. If A is a commutative complete locally m-convex algebra, one has only $\mathscr{M}_c = \mathscr{M}_0 \subset \mathscr{M}_1 \subset \mathscr{M}$, and the containments can be proper. Our goal is to investigate $\mathscr{M}$ and its relationship to $\mathscr{M}_0$; specifically (1) to give a description of $\mathscr{M}(A)$ in terms of A and $\mathscr{M}_0(A)$ which is valid for at least the class of F-algebras, (2) to determine when $\mathscr{M}(A)$ is one of the standard compactifications (Wallman, Stone-Čech) of $\mathscr{M}_0(A)$.

For many locally m-convex algebras, especially algebras of functions, one can determine $\mathscr{M}_0$. However, descriptions of $\mathscr{M}$ and its relationship to $\mathscr{M}_0$ seem to be limited to special cases; for example, Hewitt's description of $\mathscr{M}(C(X))$ [5] and Kakutani's description of $\mathscr{M}$ for the algebra of analytic functions in the unit disc [6]. We show that a commutative complete locally m-convex algebra A generates a lattice $\mathscr{L}$ on $\mathscr{M}_0$, and that if we impose a rather natural restriction on A, then $\mathscr{M}$ is the space of ultrafilters of $\mathscr{L}$. We give necessary and sufficient conditions on A in order that (1) $\mathscr{M}$ is the Wallman compactification of ($\mathscr{M}_0$, hull-kernel), (2) $\mathscr{M}$ is the Wallman compactification of ($\mathscr{M}_0$, Gelfand). In the second case, we show that $\mathscr{M} = \beta\mathscr{M}_0$ and obtain a correspondence between $\mathscr{M}_1$ and the A-realcompactification of $\mathscr{M}_0$.

We then specialize to F-algebras and show (1) F-algebras always satisfy the condition imposed in the general situation, (2) $\mathscr{M}$ is the Wallman compactification of ($\mathscr{M}_0$, hull-kernel), and (3) $\mathscr{M} = \beta\mathscr{M}_0$, whenever the algebra is regular.

1. The general case.

A locally m-convex algebra (hereafter LMC algebra) is a locally convex Hausdorff topological algebra A whose topology is given by a family of pseudonorms (submultiplicative, convex, symmetric functionals). For the basic properties of these algebras the reader is referred to [1] or [9]. In this paper we shall restrict our attention to complete algebras with identity element 1. If λ is a complex number we shall write "λ" for "$\lambda \cdot 1$".

The *structure space* of A is the set $\mathscr{M}$ of all maximal ideals of