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UNIFORM APPROXIMATION BY POLYNOMIALS WITH INTEGRAL COEFFICIENTS II

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Let A be a discrete subring of C of rank 2. Let X be a compact subset of C with transfinite diameter not less than unity or with transfinite diameter less than unity, void interior, and connected complement. In an earlier paper we characterized the complex valued functions on X which can be uniformly approximated by elements from the ring of polynomials $A[z]$. In this paper the same problem is studied where X is a compact subset of C with transfinite diameter $d(X)$ less than unity and with nonvoid interior. It is also studied for certain compact subsets of C^n where n is any positive integer. These subsets will have the property that every continuous function holomorphic on the interior is uniformly approximable by complex polynomials. A large class of sets of this type is shown to exist.

We endeavor to follow the notation and terminology of Bourbaki [2]. Throughout, X is a compact subset of C^n, $n \geqq 1$. We use the symbol z to stand for the n-tuple of complex numbers $(z_1, \cdots, z_n) \in C^n$. If R is any subring of C, $R[z]$ will denote $R[z_1, \cdots, z_n]$, the ring of polynomials in $z_1, \cdots, z_n$ with coefficients in R. When z is an element of C^n, we define $|z|$ by

$$|z| = (|z_1|^2 + \cdots + |z_n|^2)^{1/2} \geqq 0 .$$

We write

$$\sum_{k \in N^n} a_k z^k$$

to denote

$$\sum_{k_1, \cdots, k_n = 0}^{\infty} a_{k_1, k_2, \cdots, k_n} z_1^{k_1} z_2^{k_2} \cdots z_n^{k_n} .$$

(Here N stands for the nonnegative integers.)

If f is a complex valued function on X and R a subring of C, we say that f is R-approximable on X if for any $\varepsilon > 0$ there exists p in $R[z]$ such that

$$\|f - p\|_X = \sup_{z \in X} |f(z) - p(z)| < \varepsilon .$$

2. **Polynomial convexity.** If X is a compact subset of C^n, we define its polynomial hull $h(X)$ by