PACIFIC JOURNAL OF MATHEMATICS
Vol. 27, No. 2, 1968

A COUNTER-EXAMPLE TO A FIXED POINT CONJECTURE

EARL J. TAFT

Let A be a finite-dimentional commutative Jordan algebra over a field F of characteristic zero. Then we may write $A = S + N$, S a semisimple subalgebra (Wedderburn factor), N the radical of A, [5], [6]. If G is a completely reducible group of automorphisms of A, then we may choose S to be invariant under G, [4]. If G is finite, then we showed in [10] that any two such G-invariant S were conjugate via an automorphism σ of A which centralizes G and which is a product of exponentials of nilpotent inner derivations of A of the form $\sum [R_{a_i}, R_{x_i}]$, x_i in N, a_i in A, where R_a is multiplication by a in A. It was conjectured in [10] that the various elements x_i and a_i which occur in the formulation of σ could be chosen as fixed points of G. This conjecture was based on analogous fixed point results proved for associative and Lie algebras, [7], [8], [9]. However, this conjecture is false, and we present in this note a simple counter-example.

We consider three-by-three matrices over F. Denoting by e_{ij} the usual matrix units, set $e = e_{11} + e_{22}$, $f = e_{33}$ and $x = e_{31}$. Consider the Jordan algebra A with basis e, f, x and multiplication table

	e	f	x
e	$2e$	0	x
f	0	$2f$	x
x	x	x	0

.

Clearly A has a one-dimensional radical $N = Fx$, and $S(0) = Fe + Ff$ is a Wedderburn factor of A. By [2], all Wedderburn factors are isomorphic, so are spanned by two orthogonal idempotents. The only idempotents (nonzero) of A are $(e/2) + \alpha x$, $(f/2) + \beta x$, α, β in F. The only pairs of orthogonal idempotents are $(e/2) + \alpha x$, $(f/2) - \alpha x$, α in F. Hence the Wedderburn factors of A are of the form $S(\alpha) = F(e + \alpha x) + F(f - \alpha x)$, and clearly $\alpha \to S(\alpha)$ is one-to-one.

A has two types of automorphisms, as can be seen by a direct check. The first type $A(\delta, \pi)$, δ, π in F, $\pi \neq 0$, is given by: