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COMMUTATIVITY THEOREMS FOR NONASSOCIATIVE RINGS WITH A FINITE DIVISION RING HOMOMORPHIC IMAGE

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Wedderburn's Theorem, asserting that a finite associative division ring is necessarily commutative, has been extended to

THEOREM 1. **Let R be a noncommutative Jordan ring of characteristic not 2, and let I be an ideal in R such that R/I is a finite division ring of characteristic $p > 5$ with exactly q elements. Suppose that (i) I is commutative and every associator contained in the ideal generated by I^2 vanishes, and (ii) $x \equiv y \pmod I$ implies $x^q = y^q$ or both x and y commute with all elements of I. Then R is commutative.**

The object of this paper is to extend Theorem 1 in two directions. First we replace the assumption that R is a noncommutative Jordan ring by the weaker assumption that R is power-associative. Next we assume that R is a flexible power-associative ring but replace the hypothesis that every associator in the ideal generated by I^2 vanishes with the weaker assumption that I is associative. In each case we drop the assumption that R is of characteristic not 2.

The proof of Theorem 1 appears in [2].

By a noncommutative Jordan ring is meant a ring in which the associative law is replaced by the weaker identities

(1.1) $$(x, y, x) = 0\ ,$$

and

(1.2) $$(x^2, y, x) = 0\ ;$$

where the associator (a, b, c) is defined by $(a, b, c) = (ab)c - a(bc)$. A ring is flexible in case only (1.1) is assumed, and a ring is power-associative provided

(1.3) $$x^m x^n = x^{m+n}$$

holds in the ring for all positive integers m, n. It is known that a noncommutative Jordan ring of characteristic not 2 is power-associative [4], but there are flexible rings which are not power-associative. A ring R is said to be of characteristic not 2 if $2x = 0$ implies $x = 0$ in R.

2. Main results.